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THE MATHEMATICS TEACHER

DECEMBER

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NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

THE MATHEMATICS TEACHER

THE MATHEMATICS TEACHER

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Five Decades of Mathematical Reform—Evaluation and Challenge*

By William Betz
Rochester, New York

"We have already higher education; but toward the highest education there is still a long way to go."
Dr. John Rothwell Slater (1950)

The present year has occasioned numerous reviews of the march of events since the turn of the century. Is it not fitting that a similar scrutiny should occur in the field of mathematical instruction? And so, it is a pleasure to be able to call attention to the Anniversary Book, now rapidly approaching completion, of the Central Association of Science and Mathematics Teachers. It is entitled "A Half-Century of Teaching Science and Mathematics." In like manner, it is now thirty years since the National Council of Teachers of Mathematics was organized. What has been its influence, and what has it accomplished?

But looking back is not sufficient. Far more important is our attitude toward the future. What are our hopes, our plans and aspirations? Are we merely drifting, or are we reacting vigorously to existing conditions?

In any case, our starting point must be an honest examination of just what has happened to our subject during the past fifty years. Even a mere epitome of developments during that period is certain to be most revealing.

Beginning in 1902 with Professor E. H. Moore's famous address, we had, for nearly 25 years, a great flowering of interest in mathematics, of forward-looking readjustments, of hopeful reorganizations. This period culminated in the National Report of 1923. There followed a *second* phase, in the early 1920's, dominated by the ever more vexing problem of mass education, and characterized by a nationwide battle of objectives that has been continued to this day. It led to the creation of thousands of curricula in mathematics alone. A *third* phase, occupying the decade preceding the war, resulted directly from the second. Increasingly, our time-honored subject was removed from its position of prominence to that of a merely tolerated "elective." And today we are in the midst of a *fourth* period which will witness either the further elimination of mathematics or its restoration to the place it deserves. In a large number of our states, not a single hour of mathematics is now required for graduation from high school. The educational policy-makers are still pressing their demand for ever more reduced programs, for easier courses, for "functional mathe-

* Based on an address at the Tenth Summer Meeting of the National Council of Teachers of Mathematics, University of Wisconsin, August 23, 1950.

matics" devoid of algebra and geometry. Everywhere the teachers of mathematics cry out in despair that they cannot build on a vacuum. And the end is not yet. The colleges have found it impossible or inexpedient to counteract these trends. Instead, they do not hesitate to admit students without a semblance of mathematical preparation; they are offering "remedial" or introductory courses in arithmetic and secondary mathematics; and it is reported that not infrequently they even give credit for this belated kind of training.

These are some of the main facts with which we are concerned at the present moment. They have been rehearsed countless times at meetings like this. What teachers of mathematics throughout the nation are asking, with ever-increasing concern and dismay, is what an organization like this is doing about this distressing situation.

It seems clear that we are going through a severe crisis which will yield only to positive corrective steps. We are more likely to be guided in the right direction if we have a clear picture of the transforming influences that have affected our subject during the past half century. In submitting the brief summary that follows, it is the hope of the writer that it may induce others to make more detailed studies and to document further the main facts that will have to be considered.

I. The Early Reform Movement

It is not generally known today that the first major impulse toward a broad reorganization of mathematical instruction came from mathematical leaders in both European and American universities. That is, the reform movement in mathematics came "from above." An excellent source of information concerning these early efforts and developments is the First Yearbook of the National Council of Teachers of Mathematics. It contains Professor David Eugene Smith's extensive report entitled "A General Survey of the Progress of Mathematics in our High Schools in the Last Twenty-Five Years," as well as a reprint of Professor E. H. Moore's famous paper "On the Foundations of Mathematics," published originally in 1902. A perusal of these documents serves to orient the reader concerning the "era of the pioneers."

As early as 1892, the Committee of Ten, organized largely through the initiative of President Eliot of Harvard, had made some cardinal recommendations in the field of mathematics.[1] Again, the Perry movement in England, at the turn of the century, had started a rebellion against the dead formalism and backwardness which at that time characterized the teaching of mathematics practically everywhere. Professor Moore of the University of Chicago recognized the importance of Perry's ideas. He urged that the artificial separation of pure and applied mathematics be ended, that more attention be given to modern mathematics, that we build a continuous, correlated program in secondary mathematics and science, and that classroom methods be greatly improved. He expressed the hope that in this way we might succeed in arousing in the learner "a feeling that mathematics is indeed a fundamental reality of the domain of thought, and not merely a matter of symbols and arbitrary rules and conventions."

It would be a fascinating task, but one carrying us far beyond the permissible limits of this paper, to outline in similar fashion the noteworthy contributions,

[1] One of the principal sections of the Report of the Committee of Ten on Secondary Studies had to do with "Concrete Geometry." It was recommended that "the child's geometrical education should begin as early as possible; in the kindergarten, if he attends a kindergarten, or if not, in the primary school." (See the Eighth Yearbook of the National Council of Teachers of Mathematics, p. 97.) The subsequent reports of the International Commission on the Teaching of Mathematics showed that in all the leading countries systematic instruction in informal or intuitive geometry was begun not later than the seventh grade.

made during this period and later, of other mathematical thinkers on both sides of the Atlantic. A long and brilliant list of names readily suggests itself. It includes men like Klein, Jules Tannery, Enriques, Hoefler, Nunn, J. W. A. Young, George W. Myers, H. E. Slaught, E. R. Hedrick, J. W. Young, Cassius J. Keyser, and many others. The ideas which they sponsored centered around such things as the creation of a new spirit in the teaching of mathematics, an earlier emphasis on certain elements of modern mathematics, the cultivation of functional thinking, and the organization of integrated, continuous curricula.

Fortunately, it was soon realized that these necessary reforms would not result automatically from wishful thinking, but would demand the long-continued, cooperative effort of special agencies. And so, our story would now have to recite, if space permitted, the part played in this epic struggle (1) by the three associations that resulted almost directly from Professor Moore's address, (2) by the work of the International Commission on the Teaching of Mathematics, (3) by the organization of the National Council of Teachers of Mathematics, in 1920, and above all, (4) by the 1923 Report of the National Committee on Mathematical Requirements. Mention should also be made of the liberalizing influence of the College Entrance Examination Board. However, it was principally the rise of the junior high schools, beginning around 1915, which led to the confident feeling that a permanent, nationwide transformation would occur in the field of mathematics.

Throughout this period numerous attempts were made, by individual teachers and by associations and committees, to do away with the "compartment system" —a year of algebra followed by a year of geometry. It was felt that arithmetic, algebra, and geometry should be brought into closer relation with each other, and hence that mathematics should be "correlated," "fused," or "unified." But after surveying the record of the pioneering reformers, with due appreciation and gratitude, we must admit that while they accomplished some genuine improvements within the traditional framework of academic mathematics, they did not actually come to grips with the main job confronting them. As their critics pointed out relentlessly, they did not really "make out a constructive case" for the educational significance of mathematics; that is, they did not create a *compelling* type of motivation. Above all, their attempts to produce a *continuous* curriculum were regularly frustrated by the apparently impregnable organization of the four-year high school with its universal endorsement of sixteen more or less haphazard "units" as a basis for graduation.

Now, before we turn to the second and third phases of our fifty-year cycle, I should like to call attention to an unusual document, made available to American readers only recently. It is an address by none other than Professor Alfred N. Whitehead, presented many years ago in England, before a group of British teachers of mathematics. It suggests very clearly how outstanding mathematical leaders, near the turn of the century, were viewing the whole domain of mathematical instruction. A few striking passages from this trenchant address will be of interest. To quote:

> "Elementary mathematics is one of the most characteristic creations of modern thought. . . . We must conceive elementary mathematics as a subject complete in itself, to be studied for its own sake. It must be purged of every element which can only be justified by reference to a more prolonged course of study. There can be nothing more destructive of true education than to spend long hours in the acquirement of ideas and methods which lead nowhere. . . . I have had great experience with the average product of our schools as sent up to the Universities. My general conclusion is not that they have been idle at school, or have been taught carelessly. . . . But there is a widely-spread sense of boredom with the very idea of learning. I attribute this to the fact that they have been taught too many things merely in the air, things which have no coherence with any train of thought such as would naturally occur to anyone, however intellectual, who has his being in this

modern world. The whole apparatus of learning appears to them as nonsense. . . .

"There we reach one of the chief causes of the weakness of the traditional mathematical training. It is entirely out of relation to the real exhibition of the mathematical spirit in modern thought, with the result that it remained satisfied with examples which were both silly and unsystematic. Now the effect which we want to produce on our pupils is to generate a capacity to apply ideas to the concrete universe. . . . The study of algebra should commence with a systematic study of the practical application of mathematical ideas of quantity to some important subject.

"[In geometry, likewise, the curriculum] should be rigidly purged of all propositions which might appear to the student to be merely curiosities without any important bearings. . . .

"What, in a few words, is the final outcome of our thoughts? It is that the elements of mathematics should be treated as the study of a set of fundamental ideas, the importance of which the student can immediately appreciate; that every proposition and method which cannot pass this test, however important for a more advanced study, should be ruthlessly cut out; that with the time thus gained, the fundamental ideas placed before the pupils can be considerably enlarged so as to include what in essence is the method of coördinate geometry, the fundamental idea of the differential calculus in relation to rates of increase, and the geometrical notion of similarity, [as well as] some sets of social or scientific statistics, and the use of the polygon of forces in the graphical solution of mechanical problems. Again, this rough summary can be further abbreviated into *one essential principle*, namely, *simplify the details and emphasize the important principles and applications.*"[2]

II. The Battle of Objectives

The visions of the mathematical thinkers to whom we have referred were soon to be tested in a crucible of fire. As the millions poured into the new "universal" high schools, an unparalleled expansion of secondary education was on its way. We were determined to try something entirely new. In the place of the dual system of Europe, we proposed to extend unlimited educational opportunities to "all the children of all the people." And who would deny that to an amazing extent this American dream was realized? From 1900 to 1940 the enrollment figures of the four-year high schools rose from 519,251 to 6,601,444, an increase of 1171%, while the entire population changed from 76 million to nearly 132 million, an increase of 73%. To accommodate this vast throng of students, new buildings, often of palatial appearance, began to spring up from coast to coast. Nothing like it had ever happened anywhere. A new era in education had dawned which to this day remains one of humanity's most glorious hopes.

But mass education at the higher levels, greeted with such justifiable enthusiasm, proved to be a problem of staggering difficulty, as might have been expected. The road to real education has always been long and hard. From the beginning the new high school should have dedicated itself to *excellence*, for a democracy cannot be built on a foundation of mediocrity and inefficiency, nor can it safeguard an essential supply of leaders without an insistence on superior training and mastery. But instead of making secondary education a real challenge and offering it as a privilege provided at public expense, our educational policymakers only too often avoided all unpleasantness by eliminating the need of exertion. Whether they desired it or not, they actually tolerated or fostered a spirit of indolence, the very negation of genuine education. All-around happiness seemed to be their main concern. The word "failure" was ostracized.[3]

[2] The title of this address, published in England in 1912, was "Mathematics and Liberal Education." This valuable paper is now included in the collection of "Essays in Science and Philosophy," by Alfred North Whitehead, brought out by the Philosophical Library, New York, 1947, pp. 175–188.

[3] In the Convention Digest of the 29th meeting of the Representative Assembly of the National Education Association, July 3–7, 1950, in St. Louis, we find in one of the summaries of reports such statements as the following (page 4): "*The trend in promotions, especially in the West, is toward a no-failure policy.* . . . The few children who are promotion problems should be placed where they will be most happy and useful. . . . There was a strong sentiment among members of the group in favor of doing away with grade levels. *A child's individual capacity,* the group felt, *is the most effective school standard of achievement.*" The high schools seem on the point of imitating this trend. It has not been

Said one educator, "Pupils never fail, teachers fail." And so, standards were relaxed, and automatic promotion became increasingly popular. The doctrine of "*adaptation* to individual needs and interests" became the controlling gospel of education. To meet every variety of taste, ability, outlook, and whim, there occurred a vast extension of curricular offerings. The high school diploma, based on sixteen arbitrary "units," assumed the character of an attendance certificate without educational connotations. Similar developments occurred in the college field.

Some day the fantastic story of the era of curriculum revision, beginning in the early 1920's, will have to be written. The curriculum became the "whipping boy" of education. Every educational ailment was promptly attributed to "wrong curricula." Subjective factors such as aimlessness, indifference, and laziness, known to every teacher as major causes of failure, were conveniently ignored or discounted. In due time, every town and hamlet felt the urge to produce its own streamlined programs. The list of "objectives" grew by leaps and bounds. The whole educational process became a scene of confusion. As Professor Edgar W. Knight pointed out so clearly, nothing else could result from this incredible multiplicity of aims. To quote,

> "More than fifteen hundred social aims of the study of English, more than three hundred aims of arithmetic in the first six grades, and more than eight hundred generalized aims of the social studies have been listed here and there in courses of study and in special studies. In one course for the social studies in the seventh grade appeared one hundred thirty-five aims; in another subject more than eighty aims were found; the objectives of a junior-high school course were so numerous as to require many pages merely for their listing."[4]

Elementary and secondary mathematics, likewise, became the victims of the nationwide epidemic of wholesale curriculum revision. Many thousands of mathematical courses of study are now cluttering the shelves of our curriculum morgues. Very many of them were found to have no real merit.[5] Apparently, they were written under pressure, by teachers having no special training and no facilities for such a highly technical job. At a somewhat reduced rate, the fabrication of new curricula is still going on.

As we look back upon the two phases of our story which we have just reviewed, we cannot escape the force of certain major impressions. The first is that within the framework of our time-honored compartment system, of antiquated examinations and textbooks, and of fossilized classroom procedures, it was next to impossible to create the kind of new approach which the early reformers had so clearly and so urgently demanded. It was not sufficient to provide documents like the National Report of 1923. A radically new organization of our schools was imperative, including the abandonment of the four-year high school in favor of a longer exposure to the aims and processes of secondary education. We needed *continuous* curricula, to replace the fragments we were constantly offering. And we certainly needed a vastly better teacher training program. Instead, we continued the traditional machinery and constantly tried to "pour new wine into old bottles." The individual teacher was powerless to cope with this impasse. And so, there arose an almost unbridgeable chasm between the ideal demands of mathematics and the actual practices of the school. Thus it was that, step by step, the battle of conflicting objectives induced a decidedly negative and hence disastrous orientation in the field of mathematics.

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revealed how any type of cumulative work can be done effectively under such a plan. Many teachers of mathematics feel that these policies are virtually wrecking all mathematical instruction.

[4] Knight, Edgar W., *Progress and Educational Perspective*, The Macmillan Company, 1942, p. 126.

[5] See, Bruner, Herbert B. (and others), *What Our Schools Are Teaching*, Bureau of Publications, Teachers College, Columbia University, 1941.

III. The Decline of Mathematics

During the decade preceding World War II the trends and policies to which we have just referred led to the severe educational crisis from which we have not yet recovered. To be sure, there are many who would deny emphatically that such a crisis exists. Externally, in terms of enrollment figures and impressive school buildings, things seemed to be most glorious. But the teachers knew right along that under this smooth surface there lurked menacing rocks and dangerous eddies. For they were called upon to endorse and apply unconditionally a highly contradictory and unworkable set of ideas. Theorists without classroom experience seemed determined to make them gallop bravely in all directions at once. At a safe distance from the classroom, these theorists preached the simultaneous and enthusiastic acceptance of such mutually exclusive or definitely erroneous views as were embodied (1) in Dewey's pragmatic theory of knowledge, his disruptive concept of truth, his insistence on both immediate personal experience and social utility; (2) in Kilpatrick's doctrine of change, his glorification of "individual needs and interests," his rejection of organized curricula, and hence his advocacy of the planless school; and (3) in Thorndike's mechanistic psychology, with his deterministic faith in measurement and fixed IQ's, and his success in undermining basic aspects of the "transfer of training." All this was bound to cause chaos, as well as a "revaluation of all values." The traditional school subjects were stripped of their ancient position of dignity and unique worth. A course in beauty parlor work was given the same educational "credit" as a year of English or science or mathematics. Standards of achievement evaporated and automatic promotion became widely prevalent.

That this analysis is not a purely subjective one, could easily be proved by a long array of pertinent quotations. In fact, it was Dr. Dewey himself who remarked a few years ago, in a highly publicized magazine article,

"We agree that we are uncertain as to where we are going and where we want to go, and why we are doing what we do."

In like manner, an outstanding educational commentator, surveying the educational nihilism of the 1930's, had this to say:

"The crucial error is that of holding that nothing is any more important than anything else, that there can be no order of goods and no order in the intellectual realm. There is nothing central and nothing peripheral, nothing primary and nothing secondary, nothing basic and nothing superficial. *The course of study goes to pieces because there is nothing to hold it together.* Triviality, mediocrity, and vocationalism take it over because we have no standard by which to judge them. We have nothing to offer as a substitute for a sound curriculum except talk of personality, 'character,' and great teachers, the slogans of educational futilitarianism."[6]

Now let us briefly look at the fate of mathematics during this period. The following paragraphs will serve to summarize the developments in which we are here interested.

1. The "cultural" claims of mathematics were increasingly ignored. The subject had to be justified, if at all, in terms of social, utilitarian, and vocational objectives.

2. Academic mathematics became an elective subject in the majority of our states. As a result, the number of students enrolled in algebra or geometry classes soon shrank at a rapid rate. In typical high schools the number of teachers required to teach secondary mathematics was reduced to fewer than half the former figure.

[6] See the article entitled "Reminiscences," in The Mathematics Teacher, October, 1940, p. 245.

3. It became current doctrine that only a minority, "the few," had any use for the customary offerings in algebra and geometry. All the rest, "the other 85%," needed a totally different diet, the "mathematics of daily life," or the "mathematics of life situations."

4. But while these attacks were going on at the top, an even more catastrophic development occurred at the bottom of the educational ladder. A self-constituted committee of school men in Illinois announced, some 20 years ago, on the basis of scores obtained with the aid of specially constructed tests, that facts which they claimed to have established concerning "number readiness" demanded a drastic postponement of arithmetic instruction. For generations, throughout the world, pupils had learned arithmetic in the primary grades at least as well as reading and writing. But suddenly that was denied or ignored. The campaign for postponement began. It is a most unsavory story. For it turned out that the idea of postponement fell on very receptive soil. It was true that a mastery of the basic number facts could not be attained incidentally, by the route of activities and personal experience alone, nor by individual drill of a mechanistic type. That was a mortal defect. How postponement was to change that fact, was never divulged. And it has remained a mystery ever since. Nor was any attention given to the real root of the troublesome arithmetic situation, namely, that teachers either were not allowed to teach arithmetic in a systematic way, or had come from normal schools or teachers colleges that left them untrained in the field of number work.

The protagonists of postponement paid no heed to the severe criticisms of outstanding students of arithmetic and of organizations like the National Council of Teachers of Mathematics. They rode roughshod over every type of opposition. The children of America became their helpless victims.[7]

5. One evil often begets another. The consequences of postponement speedily showed themselves. The war years furnished dramatic evidence of the widespread collapse of mathematics as a school subject. Very many of the men taking the selective examinations given by the armed forces lacked even a rudimentary acquaintance with fractions and decimals. And from the high schools, throughout the nation, came a chorus of complaints that ignorance of arithmetic, at all levels, was the rule rather than the exception. Thus began the vain struggle to counteract by means of "remedial" lessons the vacuum created by the elementary school.

6. As the elementary school artificially extended the period of infancy into the upper grades, the entire mathematical program which had received such careful attention during the three preceding decades began to give way. The mathematics of the junior high school, which had been our major hope in the direction of genuine mathematical reform, fell even below the level attained in the old grammar school. Such vitally necessary and highly successful essentials as informal geometry, with its rich body of applications, and introductory work in symbolic thinking, either disappeared or were completely devitalized.

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[7] Teachers of mathematics should consider it a solemn duty to become acquainted with the authoritative literature pertaining to the postponement of arithmetic. Among the most important titles are the following:

1. Brownell, William A., "Arithmetic in Grades I and II," Duke University Press, 1941. (This is the cardinal research monograph relating to the basic factors of the controversy. It is fully documented. The final bibliography contains 60 titles.)

2. Brownell, William A., "A Critique of the Committee of Seven's Investigation on the Grade Placement of Arithmetic, Topics," *Elementary School Journal*, March, 1938, pp. 495–508.

3. Washburne, Carleton W., "The Work of the Committee of Seven on Grade-Placement in Arithmetic," *Thirty-Eighth Yearbook of the National Society for the Study of Education*, 1939, pp. 299–324.

4. Dickey, John W., "Readiness in Arithmetic," *Elementary School Journal*, April, 1940, pp. 592–598.

7. As a result of all this, the ninth school year became the storm center of the mathematical scene. Entering pupils commonly showed such wide gaps in their mathematical preparation that teachers were forced to devote a large fraction of valuable classroom time to a belated and often futile program of remedial instruction. Increasingly, it was impossible to complete a normal year's work. And so, the curriculum problems of the high school grew by leaps and bounds.

Thus it was that the war era, and the years following it, inherited a total situation with which we are still concerned from day to day.

How did our leading educational policy-makers react to these developments? The majority seemed to view these trends with complete equanimity. They did little or nothing to change them. Hence it is a rather remarkable fact that at least one well-known educator expressed a contrary opinion. He said,

"What a cock-eyed world it is in which, as the need for mathematics becomes greater and more universal, the smaller is the percentage of high school pupils studying it."[8]

However, a far more incisive protest and word of warning is contained in the official document entitled "Manpower for Research."[9] In no uncertain terms it directs attention to the vital issues of national efficiency and defense. Modern wars are fought with the weapons forged by science and mathematics. To undermine or neglect thorough training in these basic disciplines is to invite the danger of national disaster.

[8] Douglass, Harl R., "The Double-Track Plan of High School Mathematics," in The Mathematics Teacher, February, 1943, p. 58.

[9] "Manpower for Research," Volume Four of *Science and Public Policy*, A Report to the President, by John R. Steelman, Chairman, October, 1947. (Reprints may be obtained from the Superintendent of Documents, U. S. Government Printing Office, Washington 25, D. C.; price, 35¢.)

IV. Whither Mathematics?

By the time we had reached the year 1940, it had become crystal clear that we faced the alternative of either enduring the permanent elimination of mathematics as a major school enterprise or engaging in a relentless struggle for its restoration to its rightful place. It looked as though utter defeat stared us in the face, especially when the colleges accepted the situation almost without lifting an eyebrow.

Now, before we go on, let us admit frankly that it took the severe battering we received at the hands of the opposition to effect—at long last—a few of the readjustments on which the mathematical leaders had insisted for decades. It seemed almost impossible to get rid of obsolete or useless materials, to modernize classroom procedures, and to inject into the subject the spirit and the types of emphasis which Klein, Moore and Whitehead had so dramatically demanded. Above all, we began to see that we must really do something about the mathematical needs of the majority of our pupils, "the other 85%." By our slowness in putting our house in order we made it very easy for our critics to build up a strong case against us.

Today it may be reported that gratifying progress has been made, but that much remains to be done. A long road lies ahead of us, if we desire to attain victory for our cause. And it should be fairly obvious that for years to come we shall be engaged on three major battle fronts. Using the popular terminology of the moment, I shall designate them as "Operation Mathematics," "Operation Education," and "Operation Administration." It remains to sketch briefly the nature of these further engagements.

A. *"Operation Mathematics."* The Joint Commission Report of 1940 and the reports of the Commission on Post-War Plans of The National Council of Teachers of Mathematics have furnished much of the ammunition that is so vitally necessary in this major action. Let teachers of mathe-

matics study these documents again and again, as well as numerous helpful articles in our professional literature. From these publications they will derive a clearer understanding of the issues that lie ahead of us. We must have *continuous* curricula extending from the kindergarten to the junior college. Each school year must make a definite contribution toward the realization of essential mathematical goals. *Arithmetic must not be postponed. In Grades 7 and 8 of* ALL *types of schools we must provide, for* ALL *pupils of normal ability, an up-to-date program in junior high school mathematics.* Beginning in Grade 9, we should offer at least two different options, as follows: (1) a sequential course in academic mathematics, representing a sound foundation for professional and technical work, and (2) a two-year course in general mathematics for those who are interested in immediate life preparation. In all cases, classroom methods must cease to stress mere manipulation, but should give due attention to considerations of insight, mastery, and appreciation. And, from first to last, as an all-pervading theme song, the pupils should be made aware of the immensely important role of mathematics in modern life.

2. "Operation Education." Although millions of our young people are crowding our classrooms, we seem unable to make up our minds just what we are trying to accomplish. It is not a secret that countless classroom teachers feel that there is a good deal of aimless drifting. And it should be quite clear that by no stretch of the imagination can the teaching of mathematics be made to fit our dominant educational philosophy. Mathematics is a system of ideas. It is the product of the ages, the joint contribution of many thinkers. Mathematical truths are objective and relatively permanent. They do not "emerge" from day to day. Like the alphabet, they are independent of personal likes or dislikes. The mathematical curriculum cannot be derived solely from the pupil's individual experience, nor from unorganized life situations. It constitutes a *system* and must be taught as a system. This means, moreover, that mathematics is cumulative. Hence it demands progressive insight and mastery. Each step counts. It is hard to understand how even the most elementary number work could be "adapted to individual needs and interests." What can and should be "adapted" is the *learning rate*. But without standards of achievement the mathematical program soon falls apart. A teacher simply cannot handle a class of forty or more pupils in mathematics when the majority differ greatly in their mathematical background. The constant demand for repetition and endless remedial lessons does not arise in schools which give due attention from the beginning to careful teaching and to *immediate* corrective measures. Unless and until our educational policymakers admit the truth of these things, there can be no peace in the field of mathematical education, and there is absolutely no hope of recovery.

The whole picture is very much blurred by the constant assertion, in highly dramatic comparisons of the old and the new, that under the new dispensation our children show a much greater mastery of "the three R's" than in former days. One wonders from what sources the authors of such statements derive this information in the case of arithmetic. Throughout the nation the teachers of mathematics have reported exactly the opposite situation. If the educators gave some attention to these contrary statements, they might be more willing to coöperate constructively on behalf of our cause.[10]

[10] The breakdown of arithmetic in the elementary school, on a wide front, has been attested so thoroughly that it cannot be dismissed in cavalier fashion, as a mere fantasy or a distortion of facts. See, for example, Orleans, Jacob S. and Saxe, Emanuel, "An Analysis of the Arithmetic Knowledge of High School Pupils," *College of the City of New York Research Studies in Education, No. 2*, 1943.

For years the pages of THE MATHEMATICS TEACHER have offered a running commentary on this situation. The following issues will aid in

3. "Operation Administration." The individual teacher is under obligation to carry out the policies which are in force in the school system with which he or she is associated. Hence the basic reorientation which we have suggested will not occur until it has been endorsed by those in authority. In other words, until the average school administrator is willing to break away from current educational dogmas, there will be no genuine mathematical reform. And he will not change over from a course which he considers popular or desirable until he is convinced that overwhelming public opinion favors such a reversal.

If this analysis is sound, we are led to see that only a compelling type of nation-wide publicity, directed at the average parent and taxpayer, is likely to lead to the necessary changes. The crucial importance of mathematics in the education of the average pupil must be stressed. *The postponement of arithmetic must be stopped.* We must again point out the merits of the 6–3–3 or the 6-6 plan. *We must restore a genuine type of junior high school mathematics.* The universal educational chaos must be counteracted. This means that we cannot have unlimited "adaptation." Who would think of "adapting" spelling to "individual needs and interests"? *The number facts are the same for every child.* We must explain the necessity of standards, and the absurdity of automatic promotion. The world does not offer a penny for a million *wrong* answers. The pupil's subsequent professional or vocational status depends on *honest* educational achievements. The average parent will understand the cogency of such arguments, and ultimately this insight will lead to a public demand for real educational justice to our children and young people.

This urgently needed kind of publicity must come primarily from organizations like the National Council of Teachers of Mathematics. Its officers and all its members have the grave responsibility and sacred duty of sponsoring such a campaign of public re-education. In a democracy like ours that is the most effective weapon we have.

Once again, grim days have come upon us, and brave American boys have been fighting and bleeding for freedom on distant battlefields. This is not the time to argue about "mere differences of opinion." But most decidedly, it is high time that policies and practices which the rank and file of experienced teachers have found to be opposed to the highest educational ideals and to the welfare of the nation be replaced by a course that shall be more in harmony with the real interests of our young people.

If we have used a militant phraseology in these pages, it is only because all other approaches have proved futile. It would be much more pleasant, if it served a real purpose, to "talk peace." But the iron curtain that now separates us from the opponents of mathematics will not disappear until there is a disposition to reconsider the basic issues to which we have referred.

Our organization recently became affiliated with the National Education Association. This is certainly a step in the right direction. It symbolizes the earnest desire of the teachers of mathematics to coöperate with all truly progressive educational agencies. Above all, this act of affiliation should prove to be a powerful instrument in establishing a basis for mutual understanding and constructive readjustments.

A Final Word

In this brief study of the past five decades of mathematical reform we tried to give due prominence to the two major

---

creating the perspective that is essential: October, 1940; February, 1941; March, 1941; May, 1942; October, 1942; February, 1943; October, 1943; November, 1943; December, 1943; April, 1944; October, 1944; January, 1945; April, 1945; December, 1946; April, 1947; October, 1947; April, 1948; April, 1950.

factors that really determined the course of events. The first had to do with the forward-looking plans of the pioneering mathematical leaders. While the influence of their ideas is felt to this day, it cannot be denied that these men did not sufficiently sense the coming educational revolution under the impact of mass education. Their interest was centered almost exclusively upon academic mathematics. It was a fateful error that the role of mathematics in a system of universal education received so little attention at their hands.

In due time the curriculum machinery was taken over by the educators. By way of contrast they followed an exactly opposite course. Their primary concern was one of adaptation to "the needs of the many." Unfortunately, these "needs" were regularly interpreted, for all but "the few," in terms of the "minimum essentials of everyday life." The educators looked with scorn upon the claims of "academic" mathematics. The ingredients of the new "life mathematics" which they advocated rose but slightly above the level of grocery store arithmetic and some phases of what, amusingly enough, they called "consumer mathematics." Thus we entered the era of "mathematics without mathematics."

Together, the two unfortunate mistakes to which we have referred are largely responsible for our present troubles. Hence a twofold correction is now needed. On the one hand, we must insist on a sound mathematical foundation, in terms of the recognized fundamentals of basic mathematics. And we must enable all capable students to attain the maximum of mathematical training which can be achieved in the high school.[11] On the other hand, for those whose chief concern is immediate life preparation, we must provide, not a pauperized type of mathematics, but a mathematical program that shall be both vocationally significant and culturally worth while.

This two-track program would seem to be the most promising answer to the challenging demands of both the pioneering mathematical leaders and their critical opponents. Let us hope that, long before another cycle of fifty years has elapsed, the dual problem of our American system of education, that of "the many" and of "the few," will have found a more adequate solution.

[11] As long ago as 1924 the National Society for the Study of Education devoted its *Twenty-Third Yearbook, Part I*, to "The Education of Gifted Children." In recent years the Educational Policies Commission has taken particular pains to emphasize "the educational needs of individuals who have superior intellectual capacity." Two of its reports, "Education for All American Youth" (1944) and "Education for All American Children" (1948), support the idea that "the schools and colleges must give special attention to the education of their gifted students." Feeling, however, that "this policy needs further emphasis and elaboration," the Commission has just published a special monograph devoted to "Education of the Gifted" (Washington, 1950).

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Installment Buying

By L. J. Adams
Santa Monica City College, Santa Monica, California

Clifford Bell
University of California, Los Angeles, California

Teachers of arithmetic are all familiar with one or more methods of computing the simple interest rate involved in installment buying, but some may not be aware of the underlying assumptions implied in the methods they use. It is the purpose of this article to describe three commonly used methods, to actually derive the formulas involved, and, more particularly, to call attention to the assumptions made in each method. It is hoped that these discussions will give the teacher a better understanding of a rather difficult application of arithmetic.

For our discussion we should recall the following formulas involving simple interest:

$$I = Pti \tag{1}$$

$$S = P(1+ti) \tag{2}$$

where

I = simple interest
P = principal
i = interest rate
t = time in years
S = amount of P after t years if it is invested at a simple interest rate i

Furthermore,

$$D = Std \tag{3}$$

$$P = S(1-td) \tag{4}$$

where

D = simple discount
d = simple discount rate

and

P = present value of the amount S due in t years if a simple discount rate of d is used.

The three commonly used formulas for determining the simple interest rate involved in installment buying are:

$$i = \frac{24C}{n(B-C+R)} \tag{5}$$

$$i = \frac{24C}{n(n+1)R} \tag{6}$$

$$i = \frac{24C}{(n+1)B} \tag{7}$$

where

B = unpaid balance (cash price less the down payment)
n = number of monthly payments
R = monthly payment
C = carrying charge
i = simple interest rate

Usually no attempt is made to justify formula (5) when it is used in an arithmetic course. However, formulas (6) and (7) may be obtained by arguments which seem to be reasonable but which give no clue as to the actual state of affairs. We illustrate by an example.

Example. A stove sells for \$220 cash. It can be bought on the installment plan by making a down payment of \$20, followed by 24 monthly payments, including a carrying charge of 7% per year. Find the simple interest rate paid by the buyer.

Solution. Using the symbols given above, we have

$$B = 220 - 20 = \$200$$

$$n = 24$$

$$C = 0.07(200)2 = \$28$$

$$R = \frac{\$200 + \$28}{24} = \$9.50.$$

Using formula (5),

$$i=\frac{24(28)}{24(200-28+9.50)}=0.154, \text{ or } 15.4\%,$$

which gives one result found in a few textbooks.

A very large number of arithmetic texts and workbooks would solve this example as follows:

The customer owes \$228 the first month. At the end of the first month he pays \$9.50, and he therefore owes \$218.50 for the second month, and so on. A table is prepared to show the amount owed each month, and the total amount upon which interest should be computed. The table follows;

The customer owes:

\$228.00 for 1 month
\$218.50 for 1 month
\$209.00 for 1 month
.
.
.
\$9.50 for 1 month

This is equivalent to the sum of \$228.00, \$218.50, \$209.00, . . . , \$9.50 or \$2850 for one month. A total of \$28 for two years is paid for interest, hence using formula (1),

$$28=2850(\tfrac{1}{12})i$$
$$i=0.118, \text{ or } 11.8\%.$$

This same result would have been obtained by substituting in formula (6). It should be noted that the interest rate obtained is not the same as that obtained by the use of formula (5).

Other texts and workbooks would solve our illustrative example as follows. Interest should be paid on the first \$9.50 payment for one month, on the second payment for two months, and so on. But the interest on \$9.50 for two months is equal to the interest on 2(\$9.50) or \$19.00 for one month. In like manner the interest on \$9.50 for three months is equal to the interest on 3(9.50) or \$28.50 for one month, and so on.

The following table is prepared.

Number of payment	Compute interest for one month
1 st	$1\times\$9.50=\$9.50$
2nd	$2\times\$9.50=\$19.00$
3rd	$3\times\$9.50=\$28.50$
.	.
.	.
.	.
24th	$24\times\$9.50=\$228.00$

The equivalent principal for one month is the sum of \$9.50, \$19.00, \$28.50, . . . , \$228.00 or \$2850. Using formula (1),

$$28=2850(\tfrac{1}{12})i$$

or

$$i=0.118, \text{ or } 11.8\%.$$

We note that this result is identical to that obtained by the previous method, but again we can not be certain as to the actual meaning of the answer.

The usual textbook method of obtaining formula (7) is to assume that the average time $(n+1)/2$ is used for each payment, and that the unpaid balance, B, carried forward at the simple interest rate, i, for the average time is equivalent to the total carrying charge, C. Under these assumptions, we find that

$$B\,\frac{\frac{n+1}{2}}{12}\,i=C$$

which yields, after simplification,

$$i=\frac{24C}{(n+1)B},$$

which is formula (7).

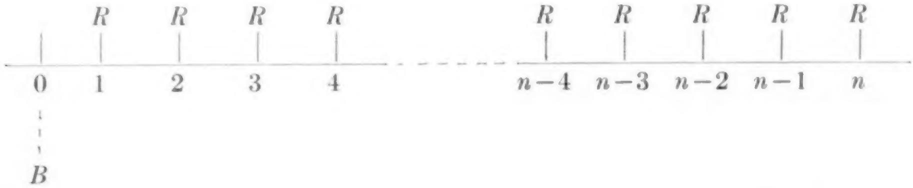
Using this formula to work the illustrative example, we have

$$i=\frac{24(28)}{25(200)}=0.134, \text{ or } 13.4\%,$$

a result which differs from that obtained previously.

Since we have obtained three different

results for the solution of our illustrative example, it appears that a careful derivation of each of the formulas (5), (6), and (7) is in order. The derivation of these formulas will be made easier by the use of the following line diagram.



This diagram indicates that the unpaid balance, B, the value of the debt at the present time (time 0 on the diagram) is to be paid off by n monthly payments of R each, where the payments are made at the ends of the months.

To derive formula (5) we assume a focal date (or comparison date) at the end of the payment interval. Making use of formula (2) we have

$$B\left(1+\frac{n}{12}\,i\right)=R\left(1+\frac{n-1}{12}\,i\right)+R\left(1+\frac{n-2}{12}\,i\right)+R\left(1+\frac{n-3}{12}\,i\right)+\cdots+R\left(1+\frac{1}{12}\,i\right)+R.$$

This equation of value states that the unpaid balance, B, carried forward at a *simple interest rate* i to the end of the payment interval is equivalent to the sum of the values of the n monthly payments of R each, where each payment is carried forward to the focal date by the use of formula (2).

Making use of the fact that the sum of the integers from 1 to n is given by the formula

$$1+2+\cdots+n=\frac{n(n+1)}{2}$$

the reader will have little difficulty in solving the above equation of value for i, obtaining

$$i=\frac{24C}{n(B-C+R)},$$

which is formula (5). Note that C was introduced in this result from the relationship $C=nR-B$.

Formula (6) may be derived as follows. Let us assume that the present time is used as the focal date, and let us use a simple discount rate, d, in place of an interest rate i. By the use of formula (4) we have

$$B=R\left(1-\frac{1}{12}\,d\right)+R\left(1-\frac{2}{12}\,d\right)+\cdots+R\left(1-\frac{n-1}{12}\,d\right)+R\left(1-\frac{n}{12}\,d\right).$$

This equation of value states that the unpaid balance, B, is equivalent to the sum of the discounted values of each of the n payments to the present time. Solving for d and simplifying, we have

$$d=\frac{24C}{n(n+1)R}\,.$$

Formula (6) is obtained from this by replacing d by i, under the assumption that a simple discount rate is approximately equal to a simple interest rate.

Formula (7) may be obtained by using a focal date $(n+1)/2$ months from the present time, together with the assumption that d is approximately equivalent to i. That is, we set up our equation of value using simple interest for all payments (including the outstanding balance) made before the focal date, and simple discount for all payments made after the focal date.

If n is even, the equation of value is

$$B\left[1+\left(\frac{n+1}{2}\right)\left(\frac{i}{12}\right)\right]$$
$$=R\left[1+\left(\frac{n+1}{2}-1\right)\left(\frac{i}{12}\right)\right]+\cdots$$
$$+R\left[1+\frac{3}{2}\left(\frac{i}{12}\right)\right]$$
$$+R\left[1+\frac{1}{2}\left(\frac{i}{12}\right)\right]$$
$$+R\left[1-\frac{1}{2}\left(\frac{d}{12}\right)\right]$$
$$+R\left[1-\frac{3}{2}\left(\frac{d}{12}\right)\right]+\cdots$$
$$+R\left[1-\left(\frac{n+1}{2}-1\right)\left(\frac{d}{12}\right)\right].$$

If n is odd, we have

$$B\left[1+\left(\frac{n+1}{2}\right)\left(\frac{i}{12}\right)\right]$$
$$=R\left[1+\left(\frac{n+1}{2}-1\right)\left(\frac{i}{12}\right)\right]+\cdots$$
$$+R\left[1+2\left(\frac{i}{12}\right)\right]$$
$$+R\left[1+1\left(\frac{i}{12}\right)\right]+R$$
$$+R\left[1-1\left(\frac{d}{12}\right)\right]$$
$$+R\left[1-2\left(\frac{d}{12}\right)\right]+\cdots$$
$$+R\left[1-\left(\frac{n+1}{2}-1\right)\left(\frac{d}{12}\right)\right].$$

If we set $d=i$ in each of these equations we obtain

$$B\left[1+\left(\frac{n+1}{2}\right)\left(\frac{i}{12}\right)\right]=nR.$$

Solving for i we have formula (7):

$$i=\frac{24C}{(n+1)B}\,.$$

In conclusion we summarize as follows:

Formula (5) gives the *actual simple interest rate* involved in installment buying provided that the focal date is chosen at the end of the payment interval (at the time of the last payment).

Formula (6) gives the *actual simple discount rate* involved in installment provided that the focal date is chosen at the beginning of the payment interval (one period before the first payment). In substituting i for d we assume that the simple discount rate is an approximation for the simple interest rate.

Formula (7) gives neither a simple interest rate nor a simple discount rate, but gives an approximation to either one. This approximation is obtained by setting $d=i$ in the equation of value obtained by taking the focal date $(n+1)/2$ months from the beginning of the payment interval.

Editor's Note

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Better Arithmetic

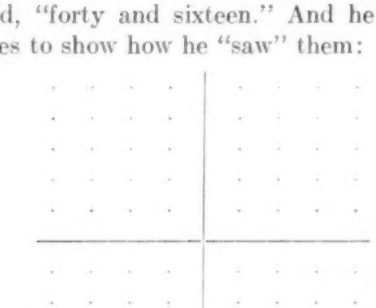
By Gladys Risden
Vermillion, Ohio

A random sampling of ten high school graduates was asked, "How many dots in the frame below?"

Three of these counted, "Two, four, six etc. . . . on to "fifty-two, fifty-four, fifty-six." One counted "One, two, three, four . . . fifty-four, fifty-five, fifty-six." Three counted the number of rows and the number in each row and thereupon gave answers of "fifty-five," "fifty-six," "fifty-seven" respectively.

One, only one, of the ten answered "fifty-six" instantly. "How did you get the answer so quickly?" we asked.

"I saw ten fours and four fours," he said, "forty and sixteen." And he drew lines to show how he "saw" them:



"And there are several other ways I could have seen '56' just as quickly: 4 eights and 3 eights, 14 less than ten eights, seven fives and seven threes."

"But I didn't learn that in school," he hastened to add. "In school I never could learn multiplication. No matter how hard I tried, I simply couldn't remember whether seven eights were fifty-four or fifty-six. I was always making little errors of one or two, calling seven fours, twenty-seven, or nine nines, eighty-two. The teachers said I was careless. I wasn't careless. I simply couldn't remember. Numbers were just so many nonsense syllables to me.

"Then after I got out of school I changed a dollar into pennies and set myself down to learn numbers. I started with threes. Two threes, six. Four threes—two threes and another two threes, six and six, twelve. Four threes as four twos and four more, twelve. Eight threes—four threes and four threes, twelve and twelve, twenty-four. Eight threes as five threes and three threes, fifteen and nine, twenty-four. Eight threes, two threes less than ten threes, etc.

" 'Twasn't long before I didn't need to see pennies or such any more. I could just think groups apart in my head. Big ones, too, like fourteen nines—ten nines and four nines, 90 and 36, 126. Twenty-six forty-twos, a fourth of four two hundreds and forty-two more.

"My kids now, they ain't having no trouble at all with multiplication. I began teaching them when they were little shavers. Three sandwiches apiece for five of us, five threes, three threes and two threes, or two threes and two threes and three more, or five twos and five more. Every one of 'em could think real problems like that out before they lost their baby teeth. Had 'em doing that instead of that 'one, two, three' stuff. Always figured that had something to do with my difficulties at school. My pop had me counting to a hundred 'fore I was four years old, saying a lot of number names that didn't mean a thing to me—and they never did mean anything 'til I took myself in hand after I got out of school."

Many children have been brought to me who are failing in school arithmetic.

Worried parents blame the children for laziness, carelessness, stubbornness and blame the teachers for not drilling enough, not being patient enough, not being firm enough, etc. I get out the milk bottle caps and we begin to group.

I have yet to find the first of these children who are failing in school arithmetic who knows what "two threes" or "five fours" means. Yet they often can answer "6" when I ask them "two times three" or write 20 when I give them '5×4'." (And quite as often they write 25 or 21 the next time I ask them "5×4.")

There was Lee who looked at three pennies and four pennies insisted there were 12 pennies. "No I can't count twelve but there has to be twelve because my teacher said three times four are twelve."

And there was Kay who said, "eight" while looking at five groups of three pennies because "my teacher told us five and three are eight."

Kay and Lee are average in I.Q. In everything, except arithmetic, they are getting along all right. Both of them could count to twenty before they were five years old. But at nine they couldn't identify "four" or "five" at a glance, couldn't see "seven" when exposed to a grouping of "three and four." Asked to select a group of twelve they count "one, two, three . . . ten, eleven, twelve" instead of the quicker way of pulling out three fours or two sixes or the like. Asked to draw ten cats they arrange them in one long line and count "one, two, three," to identify the number. They do not arrange them in groups of twos, threes, fours, or fives which would make them instantly identifiable.

We should put the preceding paragraph in past tense. It was true as of six weeks ago. But today there is a very different picture. Kay and Lee and all the others are thinking groups not ones. "Six sevens—six fives and seven more, forty-two. Or three sevens and three sevens, twenty-one and twenty-one." They do not have to think groups after the first few times. They know without thinking. But when in doubt they can check back and prove their answer.

What is wrong with school arithmetic today? Is it lack of drill? No. The children with average I.Q.'s who are brought to me for help have had drill and drill in the best tradition of drill-minded mathematicians. Is it lack of experience such as is given in the best of the progressive schools? No, it is not the lack of experience, but rather that this experience is in counting only by ones. Thus it fails to give them meaningful experience with groups. A child can count the number of absentees at his table every day until Doomsday without getting a concept of 'three.' What he needs is experience in breaking the number at his table down into threes. For example: There should be six, two threes, there. There is only one three and one more. Two are missing from one of the threes. He should get six books for the children at his table by picking up three and three more or by picking up four and two more, or five and one more. That is, he should be getting experience in seeing every number in all possible arrangements of groups and using these arrangements for instant identification and selection in the early stages and for more accurate, independent thinking in the later stages.

Ask Johnny to draw nine cats. If he draws xxxxxxxxx he'll be a memorizer, not a thinker in future arithmetic. If he draws (xxx xx xxx x) or (xxx xxx xxx) or some other grouping he is getting the foundation for thinking.

What is wrong with school arithmetic? Too many children see nine as xxxxxxxxx.

Imaginary Numbers

By John W. Cell
Professor of Mathematics, North Carolina State College, Raleigh, North Carolina

If the men of the seventeenth and eighteenth centuries could have foreseen the remarkable power of imaginary numbers, undoubtedly they would have coined some other name than the extremely unfortunate name they chose. But these numbers have been too long so named for established tradition to be changed. We can certainly eliminate the idea of unrealness of these numbers by hinting at some of their uses. In the succeeding paragraphs we shall give some problems and solutions in which imaginary numbers form an essential part of the solution. These problems are offered not to be studied in detail but to illustrate applications.

IMAGINE a force of $20+i10$ lb. moving an object along the straight line from (4,0) to (8,6) in the xy-plane, the unit of length in the xy-plane being one foot. The description of the force as $20+i10$ lb. is merely a convenient way of stating that the force has a horizontal component of 20 lb. and a vertical component of 10 lb. The imaginary number *form* states all of this succinctly. If the problem were to find the work done by the force in moving the object along that line we could proceed as follows. The angle between the direction of the force, arc tan $\frac{1}{2}$, and the direction of the straight line segment, arc tan 3/2, is easily found to be arc tan 4/7 (see Fig. 1). Then the work done by the force is given by the formula:

Work = (component of force in the direction of motion) · (distance moved)

Work = $(\sqrt{500})\ (7/\sqrt{65})\ (\sqrt{52}) = 140$ ft.-lb.,

where $7/\sqrt{65}$ is the cosine of the angle whose tangent is 4/7. We could find the amount of work more easily by finding the work which the component of the original force in the horizontal direction does in moving the object through the x-distance ($4\times 20=80$ ft.-lb.), the work which the vertical component of the force does in moving the object through the y-distance ($6\times 10=60$ ft.-lb.), and adding the two results ($80+60=140$ ft.-lb.).

IMAGINE that the usual household voltage of $120+i0$ volts is attached to an

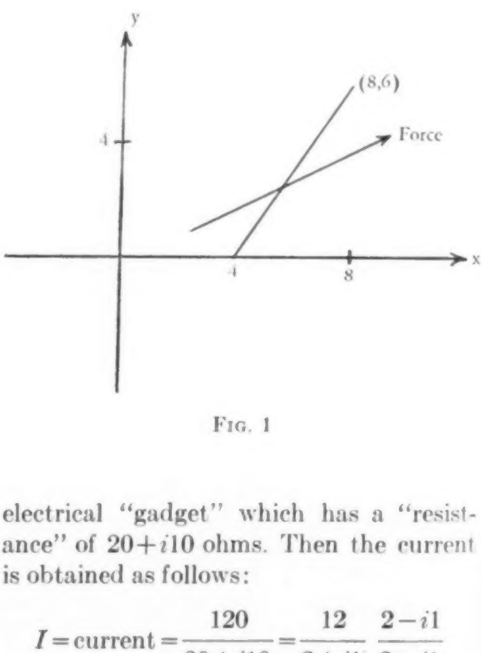


Fig. 1

electrical "gadget" which has a "resistance" of $20+i10$ ohms. Then the current is obtained as follows:

$$I = \text{current} = \frac{120}{20+i10} = \frac{12}{2+i1}\cdot\frac{2-i1}{2-i1}$$
$$= (24-i12)/5 = 4.8 - i2.4 \text{ amperes.}$$

One could describe the voltage and current as rotating vectors which rotate together and which are shown in Fig. 2 in one particular position. We notice that the current vector *lags* the voltage vector by about 26°. These two quantities could also be described in terms of sine waves in which case they would be written:

$$\text{voltage} = 170 \sin 120\pi t,$$
$$\text{current} = 7.60 \sin (120\pi t - 26.6)$$
$$= 7.60 \sin (120\pi t - 0.465),$$

where 170 is 120 multiplied by $\sqrt{2}$ and the 7.60 is obtained in a similar manner from the "length" of the current vector which is $\sqrt{4.8^2+2.4^2} \approx 5.37$.

IMAGINE a circle in the xy-plane with center at (0.2,0.5) and radius 1.3 as shown in Fig. 3). Suppose that we read the coordinates of one point on this circle, for example (1,1.52), that we compute the value of $(1+i1.52)+1/(1+i1.52)=1.30+i0.69$, that we change this last expression into the form of two coordinates (1.30, 0.69), and finally that we plot this point on a new plane (we could call it a uv-plane). If we repeat this process for a large

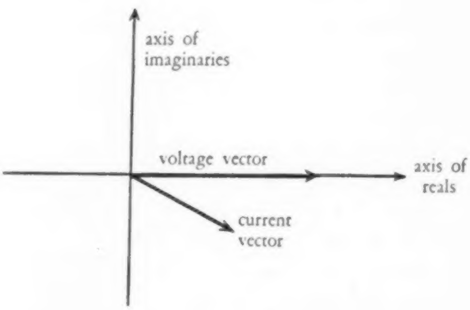


FIG. 2

number of points on the original circle in the xy-plane, each time computing $u+iv$ from $(x+iy)+1/(x+iy)$, the final result will be the curve shown in Fig. 3. This curve looks very much like a section of an airplane wing. (One can vary the thickness of the wing section and the tail angle by taking the center of the circle at different points—but the circle must always go through $(-1,0)$ for the method as described to be applicable.)

IMAGINE a weight of 32 lb. hanging from a spring which stretches 1/320 ft. for each pound of weight placed on the spring. Suppose that at time $t=0$ the weight is hanging motionless on the spring and that an oscillatory force of 100 sin $20t$ lb. begins to act on the weight. Suppose, further, that there is resistance to motion and that this resistance is always ten times the speed. We may compute a complex or imaginary quantity Z as follows:

$$Z=(ik)\left[\frac{\text{weight}}{32}(ik)+\frac{\text{resistance}}{\text{speed}}+\left(\frac{1}{ik}\right)(\text{spring value})\right],$$

where the oscillatory force is $A \sin kt$ lb. Then, for the problem as stated:

$$Z=(i20)\left[(1)(i20)+10+\frac{1}{i20}\,\frac{1}{10}\right]$$
$$\approx(i20)(i20+10)\approx -400+i200.$$

We may next assume that the force is a rotating vector defined by $F=100 \sin 20t$ and that at a particular instant this vector is horizontal and pointing toward the right (exactly as was the case for the voltage vector shown in Fig. 2). We say, then, that $F=100+i0$. We compute

$$Y=\frac{100}{Z}=\frac{100}{-400+i200}=-0.2-i0.1.$$

This last result, Y, is another rotating vector and we may think of both vectors as rotating together in a counterclockwise

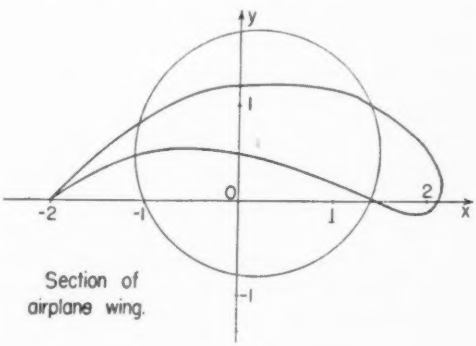


FIG. 3

direction. We may change the second vector quantity, Y, and write it as a distance or length and at an angle, i.e., we may write it in polar coordinates. We find that $Y=0.224$ at $-153.4°$. We may translate and write the formula for the displacement of the weight from the position that it would assume if hanging at rest:

$$y = 0.224 \sin (20t - 153.4°),$$

or in better form with the second portion of the angle also given in radian measure:

$$y = 0.224 \sin (20t - 2.68) \text{ ft.}$$

Any vibrating system can be studied by aid of these imaginary numbers—we have illustrated this fact for voltages and for a weight-spring problem.

IMAGINE a large table on which is placed a shallow metal tank with dimensions, for example, length = 6 feet, width = 4 feet, depth = 1 inch. Superimpose an xy-plane or grid upon this table so that the origin is at the center of the bottom of the tank and so that the x-axis is along the long dimension of the tank. Suppose that an upturned faucet is placed at (1,0) with dimensions in feet and that a small sink or outlet is at $(-1,0)$, and that the water is turned on so that the water flows evenly in the tank. The water will flow away from the upturned faucet and will drain down the sink. The path of each drop of water and the speed of each drop anywhere along its path can be studied from the standpoint of these imaginary numbers. This is a statement of what is known as a source-sink field problem and occurs in physics, electrical engineering, and aeronautics in some sort of analogy form.

Euler (1707–1783) was probably the first to discover and to use the relationship:

$$e^{i\theta} = \cos \theta + i \sin \theta,$$

which is now called Euler's formula (actually Roger Cotes discovered the logarithmic equivalent of this formula in 1714 and anticipated Euler by 34 years, but Cotes did not pursue his discovery). From this formula if we substitute $\theta = \pi$ radians, we obtain

$$e^{i\pi} = -1.$$

This is a very interesting equation involving two transcendental numbers π and e. In fact, it is this very equation that enabled Lindemann in 1882 to prove from the knowledge that e is transcendental that therefore π is likewise transcendental.

From Euler's formula we see that for $\theta = (2n+1)\pi$, with $n = \pm 1, \pm 2, \ldots,$

$$e^{i(2n+1)\pi} = -1,$$

and, consequently,

$$\log_e (-1) = i(2n+1)\pi.$$

Hence there are an infinite number of logarithms of the number minus one. Likewise it is true that there are an infinite number of logarithms of every number, except zero for which the logarithm does not exist. One logarithm of each positive real number is a real number and all of the other logarithms of the same positive real number are imaginary numbers.

Sometimes textbooks are not as correct as they should be with regard to statements about imaginary numbers. It is perfectly true that the equation $3x - 5 = 0$ cannot be solved, or has no solution,—if we have agreed that our only numbers are integers. It is likewise true that the equation $x^2 = 3$ has no solution if we have restricted our numbers to the rational numbers. It is equally true that the equation $x^2 + 1 = 0$ has no solution if we are dealing only with the so-called real numbers. But each of these statements is untrue if we enlarge the concept of number in each case. The important fact is that these imaginary numbers are just as existent and just as useful as are the so-called real numbers. There are problems in some branches of science and engineering where the associated algebraic equation is expected to possess imaginary roots which have physical importance, and the existence of a root which is a real number would be a very unsatisfactory state of affairs for these problems.

On Understanding Mathematical Methods

By H. Van Engen
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A recent article[1] in The Mathematics Teacher has focused the attention of its readers on a misconception of mathematical methods which can have serious consequences, particularly if it becomes too firmly established in the mathematics classes of the secondary schools. These misconceptions tend to crop out both in lay circles and in some professional circles. They range all the way from conceiving of mathematics as being interested only in number—hence every mathematician is an arithmetician—to more subtle misconceptions such as a conception of proof which is more rigid than can be justified by modern mathematical methods. This misconception of mathematical proof arises from a frame of mind which produces such concepts as "abolute truth" or the abolute space coordinates of Newtonian origin. Proof as used in the mathematics of the 20th Century has no such connotations.

The role played by "proof" in mathematics is crucial. This is a mere truism for mathematicians. It has a meaning not usually associated with it in the popular mind. Even the physical scientist will use techniques and chains of thought that are not acceptable to the mathematician. However, even though a proof in mathematics must meet more rigid standards than in any empirical science it does have a certain degree of flexibility not usually associated with the term in non-professional circles. When a mathematician uses the word proof it is used in connection with a set of postulates. When a mathematician says he has "proved" something he means that with his given set of postulates—it may be an implied set, one usually acceptable—and by means of the laws of logic, it can be shown that a group of theorems necessarily follow. Hence "to prove a mathematical theorem" means that in the background there is a set of postulates on which the proof has been based.

As a result of this "postulational background" it frequently happens that a given statement appears as a postulate in the postulational system A and the same statement appears as a theorem in the postulational system B. Or a definition in system A may appear as a theorem in system B. Or it may happen that in system A a theorem is "unprovable" while in system B it is provable. Proof as used in mathematics is, in this sense, a relative term. Proof is always related to, or based on, a postulational system.

The trisection problem furnishes a good analogy to illustrate the "relativeness" of proof. Every geometry teacher knows the classical problem cannot be solved. However, grant the use of a scale—instead of an unmarked straight edge—a curve, or some other device, then it is possible to trisect an angle. Here it is evident that the unprovability of a certain statement is made on the basis of a set of conditions which may be implied only because of common acceptance. For this reason the teacher should avoid automatic rejections of a student's statement that he has trisected an angle. The student may not have understood the restrictions usually imposed on angle trisectors. The automatic rejection of a proposed solution would propagate a dogmatism of a kind not possessed by mathematics.

The good teacher will strive to teach that the word proof is always used in conjunction with a set of postulates. To

[1] Fehr, Howard F., "Operations in the Systems of Positive and Negative Numbers and Zero." The Mathematics Teacher, Vol. XLII, No. 4, April, 1949, p. 171.

fail to teach this concept is to fail to teach an important element in the nature of proof. Hence, when someone claims to have proven a statement it is always necessary to inquire about his set of postulates.

This same situation pertains to the familiar rule of signs in algebra. Fehr in the article "Operations in the Systems of Positive and Negative Numbers and Zero"[2] vigorously expounds the thesis that the rule of signs cannot be proved. He cites several authoritative references, supposedly to the same effect. It is an interesting fact that one of the authorities in his impressive list (most of them not readily accessible to the high school teacher) makes this statement: "We can prove now easily the Rule of Signs. The product of two positive or two negative numbers is positive. The product of a positive and a negative number is negative."[3] Other books of mathematical repute contain much the same statement about the rule of signs.[4]

To include a reference which refutes the main thesis of a paper is, beyond doubt, confusing to the teacher not too well versed in mathematical methods. It is difficult to find any reason for doing so. Furthermore, for this teacher to find that reputable mathematicians seemingly disagree on such an elementary fact as the proof for the rule of signs is very confusing to say the least. It would seem to furnish sound evidence that mathematicians are as crazy as they are popularly supposed to be or that Bertrand Russell's definition of mathematics is literally true.[5]

In view of this situation it would be propagating misinformation of a very vicious kind to defend dogmatically the provability, or unprovability, of the rule of signs without at least warning the reader that mathematicians of first rank have published proofs for the rule of signs and have not lost their standing in the field of mathematics. Such tactics only cause confusion.

What is the explanation for this seeming disagreement among authorities? It is really very simple. Every good geometry teacher (and every good algebra teacher) lists the nature of proof as a major objective. Furthermore, good geometry teachers understand the "nature of proof" because they are successfully teaching it to high school boys and girls. Theorems can or cannot be proved depending upon the postulational system which has been accepted as the foundation of the mathematical structure under consideration.

Geometry teachers have found statements, which are given as definitions in some geometry books, given as theorems in other geometry books. This is the situation in regard to the rule of signs. It is possible to start with a postulational system which is strong enough to produce the rule of signs as a theorem. On the other hand, it is possible to build your mathematical structure on a postulational system which will not produce the rule of signs as a theorem, but the structure is completed by defining the operations with positive and negative numbers in such a way that it produces the desired mathematical structure.

Which of these procedures is correct? It is not a matter of right or wrong. Neither is it a case of being logical or illogical. Here is the point at which mathematics and aesthetics come in close contact. Mathematician A may like a given set of postulates and dislike the set of postulates set up by Mathematician B. Why? His reason may be that he simply likes his set better. Or he may favor his set of postulates because it produces neater proofs; or because it is simpler;

[2] *Ibid.*

[3] Pierpont, James, *The Theory of Functions of Real Variables*, Vol. I. Ginn and Company, New York, New York, 1905, p. 14.

[4] Fine, H. B., *The Number System of Algebra*, D. C. Heath & Co., Boston, Massachusetts, 1903.

Birkhoff, Garrett and MacLane, Saunders, *A Survey of Modern Algebra*, Macmillan Company, New York, New York, 1941.

[5] "Mathematics is the science in which we never know what we are talking about, nor whether what we say is true."

or because it has fewer postulates. There are many other reasons for preferring one set of postulates over another set.

However, in addition, Mathematician A may be a teacher. He may favor a given set of postulates for psychological reasons. It may be that his students are at a stage of mathematical maturity that makes a set of postulates, not acceptable to a mature mathematician, appropriate for the student. But it is important for B to remember that he cannot call A illogical because he does not like A's postulational system or because A's system is not a minimal set of postulates, i.e., the smallest number of postulates. If he knows mathematical methods he can object only on the basis of aesthetics or pedogogy.[6]

Fehr protests vigorously that to prove the rule of signs is illogical.[7] If his protest had been that the original article[8] on the rule of signs had been loosely written, he would have a point. The author took for granted, unjustifiably it seems, that readers would assume a postulational system which might be appropriate for use in a high school class and at the same time sufficient for proving the rule of signs. Now it seems best to state such a set of postulates. The following set of postulates might furnish a basis for the logical development of a beginning algebra course, if modified as dictated by good teaching methods.

Given: Postulational system S, with elements a, b, c, e, . . . and operations $+$ and $\cdot$ with the following properties:

1. $a+b=b+a$ and $a \cdot b=b \cdot a$
   (The operations $+$ and $\cdot$ are commutative in the system.)
2. $(a+b)+c=a+(b+c)$ and $(a \cdot b) \cdot c=a \cdot (b \cdot c)$
   (The operations $+$ and $\cdot$ are associative in the system.)
3. $a \cdot (b+c)=a \cdot b+a \cdot c$
   (The operation $\cdot$ is distributive with respect to $+$ in the system.)
4. There exist elements 0 and 1 such that $a+0=a$ and $a \cdot 1=a$
5. The system S is closed under the operations $+$ and $\cdot$. That is $a+b=c$ and $a \cdot b=c$
6. There exists for each element a of the system S an element $-a$ such that $a+(-a)=0$ (the additive inverse)
7. The cancellation law for multiplication holds, that is, if $c\neq 0$ and $c \cdot a=c \cdot b$ then $a=b$.

[6] In this connection see Young, J. W., *Fundamental Concepts of Algebra and Geometry*, The Macmillan Company, New York, 1927, p. 56.

[7] Fehr, H. F., *op. cit.*, p. 175.

[8] Van Engen, H., "Logical Approaches to $(-a)(-b)=ab$ and $X^0=1$." The Mathematics Teacher, Vol. XL, No. 4, April 1947, page 182.

The above posutlates are those of an integral domain. They hold for positive and negative integers and for polynomials. They are essentially the basic postulates with which the undergraduate works during his freshman and sophomore years. If to this set of postulates we add, for the sake of convenience, the

Definition: $a=+a$

there results a postulational system from which the theorem below logically follows.

Theorem: *The product of two positives or two negatives is a positive. The product of a positive and a negative is a negative.*

In this theorem a positive means $+a$ and a negative means $-a$. In symbols this theorem can be stated as $(+a)(+b)=+ab$ and $(-a)(-b)=+ab$ and $(-a)(+b)=(+a)(-b)=-ab$. The proof follows easily from the set of postulates. The reader interested in more details and in reading further may examine several rather elementary texts on the subject which have similar postulational systems.[9]

This set of postulates differs from that given by Fehr in his article in a fundamental way. In mathematics it is desirable to look at things in different ways, to get a better "mathematical perspective." Fehr constructed his number system and his algebra after assuming a knowledge of the natural numbers. The set of postulates given in this paper characterized the algebra which is widely known to secondary school boys and girls. Any interpretation

[9] Birkhoff-MacLane, *op. cit.*, p. 3.
Northrop, Eugene P., *Fundamental Mathematics*, The University of Chicago Bookstore, Second Edition, 1946, p. 520.

of the operations + and · and any set of elements which together satisfy the given set of postulates will be an algebra of the kind studied in the secondary schools and undergraduate colleges. Denying certain of these postulates establishes a different algebra.

This difference between constructing positive and negative numbers and characterizing an algebra is at the root of the misunderstanding in this instance. However, in neither case is there a logical inconsistency in the approach to algebra. It seems that it is impossible both to characterize and construct an algebra with the same set of postulates in an aethestically satisfying way. Since it is not possible to "have your cake and eat it too" one must make a choice on some basis. For use in the secondary school the present author has made his choice on the basis of certain psychological factors to be mentioned later.

This author will not pursue the logical aspects of using the stated postulates further. Since they have been used by mathematicians of considerable repute, further defense becomes superfluous. However, the psychological arguments for this set of postulates have not been fully discussed in the literature. These arguments deserve some attention.

The set of postulates given in this article are those usually accepted, although not explicitly stated, by authors of elementary algebra texts. It is true that in addition to accepting implicitly this set, authors frequently define $(-a)(-b)$ to be $+ab=ab$. However, the pupil's attention is not focused on the definition. This may or may not be proper and will not be discussed here. The point is that adding the definition to the set of postulates given above is superfluous—not illogical. If the teacher wishes, he can prove the rule of signs once the student has been "psychologically prepared" for the proof.

To the freshman algebra student, it is quite natural to assume that any new numbers added to the number system should obey the first five postulates of the system given above. He has been accustomed to using these postulates in his study of arithmetic in the elementary school and nothing seems more reasonable. Of course if he studies undergraduate junior-senior work in mathematics, or does graduate work in mathematics, he will find that other assumptions are more desirable because they lead more readily to important generalizations. For the high school student, however, the set S furnishes an adequate basis for the work in algebra.

Postulate 6 is the additive inverse postulate. In high school the student is led to accept positive and negative numbers by working with directed numbers on a linear scale. After having established the fact that these numbers can be given a physical interpretation, it becomes possible to get the students to accept postulate 6. Not that postulate 6 will, necessarily, be stated boldly as it has been stated here. Rather, the usual classroom procedures will devote enough time to the foundational concepts of positive and negative integers to show that, under one interpretation of addition, it is possible to find an element (integer) $-a$ such that $a+(-a)=0$.

Postulate 7 is the "familiar equals divided by equals the results are equal." This needs no comment.

The definition $a=+a$ is included because it is usually included in the high school algebra, and justifiably so, at an early stage of the work with positive and negative integers. The usual algebra book presents this definition in a side remark, such as, "from now on $+a$ will be written as a" or some other remark to the same effect.

The above is a most natural set of postulates with which it would be possible to begin the study of algebra, if algebra is to be developed as a postulational system. That it is psychologically natural to assume that the positive and negative integers obey the laws of natural numbers was a major point of the original article[10]

on the rule of signs. It is still the contention of this author that postulating the distributive, associative, and commutative laws is psychologically a good way to teach the fundamental operations with positive and negative integers. It deserves the consideration of the teachers of mathematics in the high schools.

There is yet another reason for considering a non-definitional approach to the rule of signs. Experience shows that there is reason to doubt whether the usual exercises in falling temperatures and spending money are effective as instruments for convincing the pupil that $(-a)(-b)=+ab$. These exercises usually leave an air of mystery about the rule. On the other hand, with the proper postulates, it is possible to show that the rule follows as a logical consequence. In other words $(-a)(-b)$ must be $+ab$. This in itself furnishes an excellent opportunity to teach simple mathematical methods.

In view of the extended discussion on the logic of algebra, the reader might misinterpret the author's position on the extent to which "logical approaches" can be used in algebra. It is not the author's contention that beginning algebra should be presented rigorously. This is pedagogically absurd. However, the author does feel that, traditionally, too little emphasis has been placed on the idea of proof in algebra. It seems reasonable to believe that more could be done with proof in algebra than is being done at present if present day textbooks reflect practices in the classroom. How much can be done with proof in algebra can only be determined by experimentation.

Before bringing this discussion to a close it would seem desirable to call attention to yet another criticism made by Fehr in his article. Fehr directed his efforts toward showing that a proof of the rule of signs was illogical by developing a set of postulates from which the rule of signs did not follow as a theorem. Now those who understand mathematical methods will not attempt to show logical inconsistency of one set of postulates by setting up an entirely different set of postulates, thereby expecting to conclude that the first set of postulates could not produce the rule of signs as a logical consequence. Inconsistencies in mathematical systems are exposed by showing that there is a logical contradiction involved if a particular set of postulates is to be used for the structural underpinnings of the system. Fehr did not expose any logical contradiction in the original article. He could not do so because he did not have sufficient data—he did not have the postulational system on which the original article[10] was based at the time he wrote his article. Thus, the method used to show an inconsistency is in error. Fehr seems to have been laboring under the illusion that if in one postulational system a given property is defined, then in a postulational system, in which this same property shows up as a theorem, this second system must of necessity be illogical if the first was logical. Isn't this a non-sequitor argument? Had he criticized the original article because it did not state explicitly the postulational basis for proving the rule of signs, and hence the logic was not evident, he would have had an excellent point.

It should not be presumed by the reader that this author is in complete disagreement with Fehr. It is true that "In the training of our teachers of high school mathematics there appears to be *a real need* (italicized by this author) for a careful study of the logic of algebra as well as a study of the logic of geometry."[11] This series of articles testifies eloquently to such a need. However, the teaching of the logic of algebra and geometry should of necessity include a good understanding of mathematical methods. Such courses should not degenerate into a bare scholasticism of the Middle Ages. It must be remembered that psychological factors are

[10] Van Engen, H., *op. cit.*

[11] Fehr, H. F., *op. cit.*, p. 176.

important and that it is possible to ruin the interest of the high school pupil by being too inflexible in the logical approach to a subject. Furthermore it would be an educational crime to sacrifice the best intersts of the pupil by adopting a minimal set of postulates. To be a teacher not only demands a knowledge and appreciation of the logical, but it also demands a knowledge and appreciation of psychological factors influencing the behavior of people. The good teacher teaches as much logic as can be effectively taught at the present level of maturity of the students. The good teacher will not get caught in a quibble about minimal sets of postulates when the intellectual life of the pupils is at stake.

And now as Alice and her most interesting companions so eloquently said some years ago:

"And how many hours a day did you do lessons?" said Alice, in a hurry to change the subject.

"Ten hours the first day," said the Mock Turtle; "nine the next, and so on."

"What a curious plan!" exclaimed Alice.

"That's the reason they're called lessons," the Gryphon remarked, "because they lessen from day to day."

This was quite a new idea to Alice, and she thought it over a little before she made her next remark. "Then the eleventh day must have been a holiday?"

"Of course it was," said the Mock Turtle.

"And how did you manage the twelfth?" Alice went on eagerly.

"That's enough about lessons," the Gryphon interrupted in a very decided tone; "tell her something about the games now."

NOTES ON THE HISTORY OF MATHEMATICS

Edited by VERA SANFORD
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The Chinese Abacus

By YEN YI-YÜN
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EDITOR'S NOTE: It seems appropriate to supplement the description of the loose counter abacus (THE MATHEMATICS TEACHER, November, 1950) with an account of computation with the Chinese instrument known as the *suan-pan* or Chinese abacus. Miss Yen's article will suggest a number of important points of difference between computation using pencil and paper and computation using the Chinese abacus, and while the processes of multiplication and division seem involved and complicated, the study of these techniques is of considerable interest. The Japanese abacus closely resembles the Chinese one from which it is probably derived. An account of the operation of this abacus may be found in *A History of Japanese Mathematics*, by David Eugene Smith and Yoshio Mikami, published by the Open Court Publishing Company, Chicago, 1914.

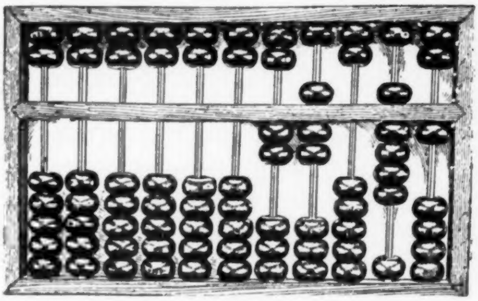
After this paper was prepared, notice was received of the American-made Chinese abacus for school use and of pamphlets describing its operation put out by W. D. Loy under the name of Loy's Chinese Calculator, 1317 Rhode Island Ave., N. E., Washington 18, D. C.

THE Chinese abacus or suan-pan is familiar to many Americans who know it as an aid to computation and who have heard that a skillful operator can manipulate it with astonishing rapidity even at times surpassing the performance of experts with electrically driven computing machines. A contest between a computer trained in the use of the Japanese abacus

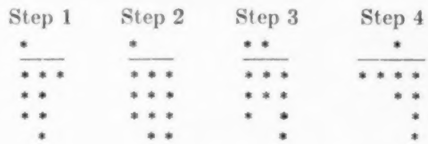
which is derived from and is similar to the Chinese one was reported in the issue of *Time* for November 25, 1946. In this instance, sums of fifty numbers of four to six figures each were found more rapidly on the Japanese abacus than on an adding machine. Comparable results have been found when the Chinese abacus was used. It is the purpose of this article to show the details of the operation of the Chinese abacus so that the reader may appreciate the differences between this and the methods to which he is accustomed both from the point of view of the learning of a process and the manipulation of the abacus. It should be noted here that Chinese children are obliged to learn two methods of computation. They must be able to compute with pencil and paper if they are preparing for the examinations to enter the secondary schools, and they must be able to compute with the abacus if they go into business.

The accompanying picture shows a Chinese abacus. It is a counting frame with beads sliding on wires or on rods of wood or bamboo. A wooden bar divides the abacus into two parts and each wire has two beads above the bar and five below. In operation, the Chinese abacus is placed flat on a table with the crossbar parallel to the table edge toward the operator. The beads are moved with the fingers and thumb of the right hand and the computer follows the written numbers he is adding with his left. A number is indicated by sliding the beads toward the bar. The two beads above the bar count as five units each, the five beads below the bar stand for ones. Any one of the wires may be designated as the units wire, the next one to the left is the tens, the next the hundreds, etc. In the illustration, if the wire at the extreme right is considered as the units wire, the number represented is 27091. A little practice is all that is needed to learn to indicate any number correctly or to read a number indicated by the beads on the abacus.

Unlike computation with pencil and paper, it is not necessary to learn separate addition combinations,—it is necessary only to learn how to add 2, how to add 3 etc. You may add 2 by adding two 1's, by adding 5 and taking away 3, or by adding 10 and taking away 8. In the accompanying diagram, the task is the addition of 333 to 841. In step 1, the number 841 is indicated with the position of the bar and the beads that show the number, the inactive beads are not shown. In step 2, 3 is added by the expedient of adding three 1's to the wire that represents units. In step 3, 30 is added by adding 50 and subtracting 20. In step 4, 300 is added by adding 1000 and subtracting 500 and 200. The sum is 1174.



THE CHINESE ABACUS
(Reproduced from *Numbers and Numerals.*)



It will be noticed that the difference between this method and computation with a pencil lies in the fact that with the aid of the abacus, you simply decide which of the three ways of adding a number applies, —you do not need to memorize addition combinations.

The same method applies to subtraction. Thus to subtract 8, you may subtract a 5 and three 1's directly, or you may subtract 10 and add 2, or you may subtract 10, add 5 and subtract 3.

In multiplication, however, it is necessary to learn the multiplication tables, but the task is simplified by omitting the

reverses. Thus the multiplication table for 2 extends from 2×2 to 2×9, but the table for 3 begins with 3×3 for 2×3 has already been learned. In multiplication, the multiplier is indicated at the left of the abacus and the multiplicand is entered toward the right, but leaving a wire free at the extreme right for each figure in the multiplier. Then the two units figures are multiplied and the product is entered on the right hand wires. Next the units digit of the multiplicand is multiplied by the tens digit of the multiplier and the result is added to the number on the second wire (i.e., the ten's wire of the product). In the case of a two digit multiplier, there is no further need for the units digit of the multiplicand and accordingly it is removed, leaving a vacant wire for the next number in the product. This may be illustrated by finding the product of 47 and 28. Using zeros to represent empty wires, the abacus would read as follows:

Indicating multiplicand and multiplier,
47000000000002800
Multiplying 8 by 7, this becomes
47000000000002856
(numbers that are part of the product are underscored)
Multiplying 8 by 40, and removing the 8 which is no longer needed, we have
47000000000002376
Multiplying 20 by 7 and adding
47000000000002516
Multiplying 20 by 40 and removing the 200, this becomes
47000000000001316

Division is more difficult. The table for dividing by 7 is translated as follows:

7 into 1, add 3 to the next right string
7 into 2, add 6 to the next right string
7 into 3 is 4 and 2 left over
7 into 4 is 5 and 5 left over
7 into 5 is 7 and 1 left over
7 into 6 is 8 and 4 left over
when you see 7 put 1 on the left string
when you see 14, put 2 on the next left string

This table is more intelligible if it is remembered that 7 into 1 means 7 into 10 yields 1 and 3 over; 7 into 30 yields 4 and 2 left over. The divisor is entered on the left, the dividend on the right and the quotient gradually replaces the dividend. Thus to divide 14762 by 7, the work is as follows:

Starting with
70000014762
When you see 14, put 2 on the left string
70000020762
When you see 7, put 1 on the left string
70000021062
7 into 6 is 8 and 4 left over
70000021086
Thus the quotient is 2108 and the remainder is 6.

To divide 36 by 12, you first divide 10 into 30 and then divide 2 into the remainder. Thus

120000036
120000306
120000300

The division tables are such that at times the quotient figure given by the tables requires alteration as the work proceeds. In dividing 448 by 64, the steps are:

Given
6400000448
Divide 60 into 400 getting 6 and 40 left over
6400000688
Divide 60 into 80 getting 1 and 20 left over
6400000728
7 times 4 is 28
6400000700

It is evident that the Chinese abacus may readily be extended to include decimal fractions and that the only precaution is to be certain just which wire represents units. It is clear too that the Chinese abacus is a device to enable the computer to indicate his partial sums, differences, products or quotients at each step of the work without perhaps consciously identifying their value. It cannot properly be called an "adding machine" because there are no mechanical parts. The abacus makes pencil and paper unnecessary until the final amount has been reached and needs to be recorded, but the computation is accomplished by the skill of the individual who uses the device.

DEVICES FOR A MATHEMATICS LABORATORY

Edited by EMIL J. BERGER
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This section is being published as an avenue through which teachers of mathematics can share favorite learning aids. Readers are invited to send in descriptions and drawings of devices which they have found particularly helpful in their teaching experience. Send all communications concerning Mathematics Laboratory Projects to Emil J. Berger, Monroe High School, 810 Palace Avenue, St. Paul, Minnesota.

Elliptical Billiard Board**

AN INTERESTING project for second course algebra students in the high school is the construction of an elliptical billiard board. The actual construction will provide concrete experience with the terminology of the ellipse and the locus definition of the curve. The completed device can be used to demonstrate the property that anything starting from one focus is reflected back through the other focus. This is the principle involved in the construction of whispering galleries.[1]

Materials needed to make this device include one piece of 5/8″ board or plywood 12″×18″, one strip of heavy tagboard 2″×48″, one box of carpet tacks, one small can of lacquer or enamel, one small roll of painters' masking tape, and two marbles 3/4″ in diameter.

First step in the construction of the device is the drawing of the ellipse on the piece of board. See Figure 1. For the 12″×18″ board suggested here, the foci, F and F', should be 13″ apart. Drive shingle nails half-way down at these two points. A convenient major axis for this particular board is 17″. Thus an ellipse which will fit the board nicely can be drawn by using a loop of cord whose total length is 30″. Place the loop over the two shingle nails, pull the loop taut with a pencil and proceed to draw the figure always keeping the string taut. The completed ellipse will have a minor axis

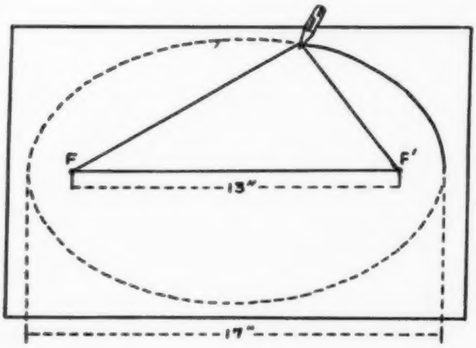

FIG. 1

slightly under 11″ in length. This measurement can be used to check the accuracy of the drawing.

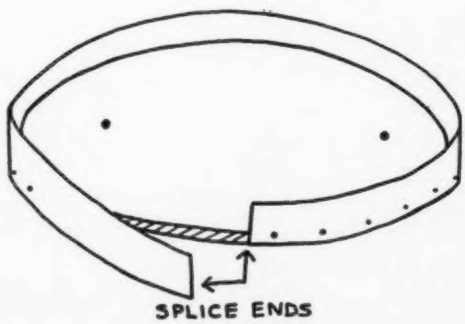

FIG. 2

* Drawings for publication in this section were made by Patricia McGroder, Macalester College, St. Paul, Minnesota.

** Plans for construction of the Elliptical Billiard Board was developed in The Mathematics Laboratory, Monroe High School, St. Paul, Minnesota.

[1] Jones, Philip S., "Mathematical Demonstrations and Exhibits," *The National Council of Teachers of Mathematics Eighteenth Yearbook*, Bureau of Publications, Teachers College, Columbia University, 1945, New York, p. 90.

Next step in the construction is to cut the ellipse from the board. For best results a jig-saw should be used, but good results can also be obtained by using a coping saw. The heavy tagboard strip can now be nailed around the elliptical board. See Figure 2. Nails should be spaced about 3/4″ apart. The exact length of the tagboard strip should be determined only after the nailing is nearly completed. Do not overlap the strip at the joining point, but cut it to the exact perimeter length with a razor blade, and splice the two ends with a piece of painters' masking tape. To add stiffness to the tagboard surface, apply several coats of enamel or lacquer both inside and outside.

To demonstrate the reflection property described above, remove the two shingle nails from the foci and place a marble at each point. If one marble is given a sharp blow it will be reflected from any point of the tagboard surface and strike the other marble.

An Angle-Board for a Circle

Purpose of the Angle-Board

The Angle-Board may be used as an inductive approach to the proofs concerning the measurement of angles by arcs of a circle, or as a practice device with problems involving the measurement of angles by arcs.

Materials Needed for Construction

1 piece masonite (12″×12″)
2 pieces yellow cardboard (12″×12″)
1 piece lucite (12″×12″)
1 sheet circular navigation paper
15 blue letters
30 eyelets
15 brass bolts ($\frac{3}{4}$″ long)
15 nuts to fit bolts
1 box colored rubber binders
1 roll blue tape

Directions for Construction

(1) Cut the masonite, cardboard, and lucite according to the dimensions listed above.

(2) Paste the circular navigation paper on one piece of cardboard. Choose at least fifteen points on this cardboard—some points around the navigation paper and some points outside of it. These points, if connected with lines, should provide illustrations of the seven kinds of angles on a circle.

(3) Paste a blue letter at each of the fifteen points chosen.

(4) Place the unmarked cardboard, the masonite, the marked cardboard, and the lucite on top of each other in the order named, and bind them together with blue tape.

(5) Drill holes through the four-ply board at each of the fifteen points.

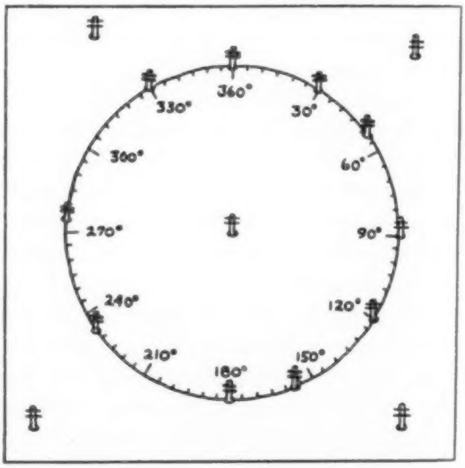

Fig. 3

(6) Place two eyelets on each bolt and insert them in the holes. Use the nuts on the protruding bolts on the back side of the board.

(7) Place the different colored rubber binders over the eyelets and form any of the desired angles.

Amelia Richardson,
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AIDS TO TEACHING

Edited by

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CHARTS

C.16—It Makes Solid Sense

Scott, Foresman and Co., 114 East 23 Street, New York 10, N.Y.
Chart; 24″×34″; Free.

Description: Printed with brown ink on ivory-white paper is this chart which is attractive, although composed of geometric shapes alone. The simplicity and directness of the pattern made by these figures compel your attention from across the room. Lay-out patterns are given for seven common solid figures in terms of their surfaces: five rectilinear figures—cube, triangular pyramid, rectangular pyramid, triangular prism, and hexagonal prism; and two curvilinear figures—cone and cylinder. Each pattern is of large size, but is not identified by name. In the title space at the top of the page are solid figures portrayed with the names attached and the comment is made in the title, "It makes solid sense if you can identify these flattened out solids!"

The back of the chart consists of advertising for "*Your Mathematics*" by Hawkins and Tate.

Appraisal: There are several activities which might stem from this chart. The identification of the solid figures from their lay-out patterns and vice versa is the simplest. The encouragement to construct similar patterns and close them up into solid models, with the consequent development of three-dimensional space perception, and of mechanical accuracy, are also worthwhile. The final activity of derivation of formulas for areas and volumes and subsequent evaluation is simplified by using this chart.

The direct usefulness of this chart as an aid in teaching pure mathematics is its most important feature. Other charts showing applications of mathematics are much needed; but for the purpose it has, this chart is simple and useful.

C. 17—Decimal Equivalents

C. 18—Lumber Table

Monroe Calculating Machine Company, Inc., Orange, New Jersey.

Each chart 7″×11″; free.

Description of C. 17: One side of this chart has decimal equivalents to six decimal places of eighths, sixteenths, thirty-seconds, and sixty-fourths. On the other side are equivalents to five places of sixths, twelfths, twenty-fourths, thirty-seconds and also avoirdupois and troy ounces as decimal parts of a pound.

Description of C. 18: This chart enables one to find the number of board feet by entering the table with the thickness and width, in inches, and the length, in feet.

Appraisal of C. 17 and C. 18: The first of these tables would be very useful for each child to have in doing his work. Certainly one should know by heart some of the decimal equivalents of fractions which occur often, but how many of us know, or need to know, all on this table—especially to five or six places! It could serve as an exercise table by having the pupils convert certain fractions to decimals and then checking the answers with the given values.

The second chart is specialized and will have little general use in most mathematics classes. However, the fact that such computations are used by lumbermen and builders enough to warrant such a table impresses pupils with the prev-

alence of arithmetic in common occupations. Discussing together the method of computing such a table and the meaning of "board feet" should be a broadening experience for teacher and pupils.

FILMS

F. 57—Vernier

Castle Films, Division of United World Films, Inc., 135 S. LaSalle St., Chicago 3, Illinois. 30 Rockefeller Plaza, New York 20, N. Y. Russ Building, San Francisco 4, California.

Technical Consultant, J. W. Barritt; 16 mm. film; 675 feet, 19 minutes; 1942; Black and white, $30.74.

Description: The film is No. 4 in a series put out by the Office of Education on Measurement. There is an accompanying film-strip which may be purchased for one dollar. The film is a study, largely in animation, of the principles of the Vernier scale and its application to precision measuring. The film could be used to good advantage in a high school physics class, a class in shop mathematics, industrial arts class or in any class in high school and college in which precision instruments are used or studied. The reviewer used the film in a college "Mathematics of Measurements" class to good advantage.

The film begins with an actual scene in which the vernier on a micrometer is used to make precision measurements in the metal working industry. This is followed by animated scenes to illustrate a 5 part and 10 part vernier scale (i.e. 5 units on the vernier scale equal 9 on the main scale). The fundamental principles of the vernier are reiterated several times and enlarged diagrams are used for clearness when measuring units are small. The film stresses the fact that results can be no more accurate than the measuring instrument used. Several problems in measurement are worked using the micrometer. The last part of the film is devoted to animated and real life scenes in measuring inside and outside diameters with a steel caliper equipped with a vernier which uses a 25 part scale.

Appraisal: The material was well presented, beginning with a very simple vernier scale and progressing to more difficult verniers. By means of animated diagrams and arrows, the fundamental concepts and reading of a vernier are clearly presented. It is consistent with the concept that measurement is only approximate and only as accurate as the instrument used. The film is best suited for a class in shop work as it does not include any other verniers such as those found on transits, sextants or stadia rods as used in surveying. Since the basic ideas are identical, the film is suited for any class which may delve into the idea of measurement (high school or college level). The film could be used as an introduction to verniers or as a means of motivating the student. I have taught the vernier to college students with and without the film and found the film to be a very helpful aid.

From a technical standpoint, the sound and pictures were very well integrated, diagrams and explanations were very clear, sufficient close ups and sequence of scenes well planned. (Reviewed by George L. Keppers, Mathematics Department, Iowa State Teachers College, Cedar Falls, Iowa.)

INSTRUMENTS

I. 27—Protractor Circle Set (P 203)

The Mathaids Company, 204 Forman Avenue, Syracuse, New York. $5.00.

Description: The set consists of a blackboard protractor and three circles whose diameters are $3\frac{1}{2}''$, $7''$, and $14''$. Each device is cut from Duron and seems to be extremely durable.

Protractor: The semi-circular protractor has a 21-inch diameter. Its surface is coated with black "Liquislate." Angular measurements are calibrated in degrees and the graduations numbered both clockwise and counterclockwise. The calibra-

tions are printed in bright yellow and their visibility against the black background is excellent. The base of the protractor contains a 16-inch ruler which serves as a convenient straight edge. A wooden handle facilitates manipulating the instrument.

Circles: The three circles are likewise coated with "Liquislate." Each has one quadrant marked off in yellow; the quadrant is subdivided into one-inch squares. Incomplete squares along the circumferences have their areas (in square inches) recorded. Thus, the students may determine the total area by finding the sum of the tiny squares of each quadrant and by multiplying the result by four; this method may be used to validate results obtained by means of the formula, $A=\pi r^2$.

Appraisal: The circles have been designed to serve several purposes; they may be rolled along a plane surface to find the circumference (this may also be found with string) and this dimension divided by the diameter to derive the approximate value of π. They may, in this way be used to develop the formula for determining the circumference of a circle, and may also, as is mentioned above, be used to validate the formula for the area of a circle. It is not desirable, however, to develop the area formula through use of the circle for no mention is made of the means used to find the areas of the incomplete squares bordering on the edges. Although the teacher may find that students accept the attractive appearance of the circles as extrinsic motivation, it does not appear that this work is superior to measuring cylindrical cans, bottles, etc. which are available in every school.

The protractor is an excellent blackboard drawing instrument because of its large size and the superior visibility of its markings. As a whole, the set is quite expensive. It seems a shame that no provisions are made at the present time for purchasing the protractor alone. (Reviewed by Bernard Singer, Hyannis, Massachusetts.)

MODELS

M. 14—Model-Math Square Root Demonstration Kit

Howard L. Wellbaum, P.O. Box #35, West Milton, Ohio.

Cardboard model; 11″×17″; 1949; $1.00.

Description: One cuts the lightweight cardboard model into squares and rectangles and then manipulates them according to the instructions to illustrate the algorithm for finding the square root of a number. A four-page leaflet ($8\frac{1}{2}''\times 12''$) describes the process very carefully, step by step.

Appraisal: The only question which a teacher need ask is this, "Do I wish to teach the algorithm for square root?" If the subject is taught, there is no better, nor simpler way to illustrate the reasons for "dividing by twice the square-root already obtained" and to give meaning to the order of steps in the whole process. Once teacher and pupils see the principles involved in this model it should be easy to make others which demonstrate the process for other numbers. It would also be wise for the teacher to construct a larger model to use in front of the class. It might even be suitable teaching procedure to demonstrate with a large model and then have each pupil make his own smaller model, using a different number of which to find the root. Other teachers who feel that the activity is not worth a long period of time, may prefer to place one of these printed, cut-out models in the hands of each pupil.

The careful explanation of the algorithm in the booklet is alone worth the cost of the device.

RESEARCH IN MATHEMATICS EDUCATION

Edited by John J. Kinsella
School of Education, New York University, New York 3, New York

In mathematics we do not accept a theory or a generalization merely because it seems to be correct in a small number of cases. We insist that a proof be given that applies to all situations which conform to the given conditions.

In mathematics education we cannot be as stringent or as rigorous in our demands. Education is an inexact art, not a highly precise study like physics. We are usually content to accept practices and procedures which have a high probability of being effective in a large variety of cases. On the other hand, we are not consistent with our ideal of proof if we assume that the reported experience of one teacher with one group of students over a limited period of time in a special school situation is a sufficient basis for advocating a revolution in the teaching of mathematics. It may be a good "pilot study" but should be followed by similar experiments which tend to affirm or negate the findings of the original study.

Our effectiveness as teachers of mathematics depends on the faith we have in the value of the procedures we use to encourage learning. This confidence tends to increase as we find other colleagues giving supporting evidence of the worth of our actions. The purpose of this section of our publication is to report that kind of corroboration as well as to throw doubt on the success of certain other proposals.

The quality of this reporting will be contingent upon the contributions of all teachers of mathematics. No one individual can be cognizant of all the significant research that is being done in mathematics education. You should not only keep us informed but offer suggestions for making the content and method of reporting research of maximum value to you.

We probably will have a tendency to report what seems new or novel, especially in our own country. On the other hand, there is frequently unpanned "gold" in the rivers of the past and in countries outside of our own. We will not succeed in underpinning the learning and teaching of mathematics with strong foundations unless we make use of all of our resources.

The Question: Does the use of visual aids lead to more effective learning of geometry?

The Study: Johnson, Donovan A. *An Experimental Study of the Relative Effectiveness of Certain Visual Aids in Plane Geometry.* University of Minnesota. December, 1948.

During 1947–48 Dr. Johnson sought some answers to the above question in an experiment involving nearly 30 classes, 12 teachers and 12 schools of varying types in Minnesota. Aware of the growing use of these aids by teachers of mathematics during the past ten years, he had searched in vain for a controlled experiment that would test the educational value of films, filmstrips and certain other visual devices in achieving the outcomes desired in geometry classes.

In plane geometry, he selected units entitled "Circles" and "Loci" for which both films and filmstrips, having similar content, were available commercially. In solid geometry, he tried to determine the value of using the Multi-Model Geometric Construction Set over a period of one school year.

The experiment was carefully designed;

the data were subjected to some of the newest and most rigorous statistical procedures. Experimental groups were shown sound films or filmstrips or both as supplements to the usual instruction. In the Control groups an amount of time equal to the visual aids time of the Experimental groups was spent in discussion or supervised study. Otherwise, the two groups followed similar class programs.

Each teacher taught both an Experimental and a Control group. The teachers in the Experimental classes were provided with a manual. This was necessary not only for standardizing general class procedure but as an aid to several of the teachers who had never taught with films and filmstrips before.

For both the "Circles" and "Loci" units, achievement tests of informational learning, problem solving, skills and ability to make applications, given at the beginning and end of each unit, were used to determine the gains in learning of the Experimental and Control groups. In addition, the same tests were given two months after the completion of the units to determine the amount of learning retained.

Some classes used both sound films and filmstrips; others used sound films alone while still others used filmstrips alone. The crux of the study was to determine the effectiveness of the four different procedures in bringing about gains in the four kinds of achievement listed above.

Findings

The quality of this experiment is not lessened by Dr. Johnson's conclusion that the results of his study were not conclusive. The results seem to depend upon the unit being studied and the type of achievement being considered, as well as the teacher and class. In the "Circles" unit the classes using both the sound film and filmstrips showed gains in the ability to apply and in the retention of information, problem-solving ability and application ability that exceeded the gains of the control groups by amounts that were statistically significant. The classes using the filmstrips alone were superior to those using sound films in the retention of information and problem-solving by statistically significant amounts, too. In the "Locus" unit, the Control group using ordinary classroom procedures surpassed the filmstrip group in construction skills by similar amounts. In neither unit did the comparison of the groups using the sound films alone and the Control groups give conclusive results in more than one class; the same conclusion is practically correct in comparing the filmstrip groups and the control groups. Likewise, the solid geometry group using the Multi-Model Set had no statistically significant advantage over its Control group. Despite these conflicting results both the students and the teachers favored the type of teaching involving the visual aids in geometry.

In concluding Dr. Johnson expressed the judgment that the results may indicate that the place of visual aids is not to duplicate what is done in the usual class in geometry but, in a supplementary way, to provide experiences not possible or feasible through ordinary classroom teaching. This writer would like to observe, too, that failure to establish stringent, statistical criteria of the 5% and 1% level type does not prove that the advantages exhibited by certain methods are of minor *education significance*.

The Question: Can significant improvements in a student's general critical thinking ability be achieved in a plane geometry class?

The Study: Lewis, Harry. *An Experiment in Developing Critical Thinking Through the Teaching of Plane Demonstrative Geometry*. New York University. 1950.

In 1941 Edward M. Glaser wrote: "Persons who have acquired a disposition to want evidence for beliefs, and who have acquired an attitude of reasonableness have also acquired something of a way of

life which makes for more considerate and humane relationships among men."[1] Teachers of mathematics will seldom dispute the proposition that the majority of students in plane geometry classes learn to think more critically about plane geometry. However, they would be more hesitant in claiming that *general* critical thinking ability is developed through plane geometry *as ordinarily taught*. Those who did make the claim would probably point to the experiments of Fawcett,[2] Ulmer[3] and Gadske.[4] Of course, in these three instances the teaching was *not* ordinary.

Dr. Lewis' study was designed to further check the findings of these three men by using teaching procedures closely resembling theirs, to strengthen the statistical bases of the findings and to provide a broader and more systematic type of evaluation. In a large high school of a city of about 400,000 he and another teacher of comparable training and ability used an experimental class of 22, a "passing control" class of 35 and a "failing control" group of 21. The experimental class used no textbook and experienced a course organized about the following topics: need of clear definitions, need of assumptions, direct deductive proof involving syllogisms, converses, contrapositives, etc., indirect proof, induction, and the interpretation of data. All but the first of these were introduced through geometric material. Then the logical ideas developed were applied to non-mathematical materials and situations, such as school happenings and controversies, advertising, news reports, newspaper and magazine articles, selections from the history of science and others. In the control classes the sequence followed was the one found in many textbooks. The emphases were on the application of reasoning principles to geometric problems and to methods of attack on geometric "originals." The fact that geometry is not the only illustration of the use of general principles of good reasoning was not brought out.

At the beginning and end of the school year all three groups were given tests of mental ability and reading, five of the Watson-Glaser Tests of Critical Thinking and the Cooperative Interpretation of Data Test. In the experimental group the students wrote monthly on a topic which they felt had a bearing on reasoning as discussed in class. This same class answered also, in anonymous fashion, a questionaire constructed to find out where and to what extent the course functioned in other areas of school living and in situations and problems outside of school living. Other teachers in the school also evaluated the effect of the course on the experimental group.

Findings

Despite the fact that there was no statistically significant difference between the means of the experimental and "passing control" groups at the beginning of the year, the gains in critical thinking over the school year made by the experimental group exceeded those of the comparison group by a statistically significant amount. On the other hand, there was no significant difference in the gains of the "passing control" and "failing control" groups. This would seem to mean that geometry as ordinarily taught does little to improve general critical thinking ability. Furthermore, the difference in the amount of geometry learned by the experimental and "passing control" groups was too small to be statistically significant. In addition, a comparison of the early and later monthly reports written by the experimental class showed considerable growth in variety of

[1] Glaser, Edward M. *An Experiment in the Development of Critical Thinking*. P. 6. Teachers College, Columbia University, New York. 1941.

[2] Fawcett, Harold P. *The Nature of Proof.* Teachers College, Columbia University, New York. 1938.

[3] Ulmer, Gilbert. "Teaching Geometry to Cultivate Reflective Thinking." *Journal of Experimental Ed.* Vol. 8, pp. 18–25. 1939.

[4] Gadske, Richard E. *Demonstrative Geometry as a Means of Improving Critical Thinking.* Northwestern University, Evanston, Ill. 1940.

application of the logical ideas acquired and in the depth of logical analysis of them. The questionnaire showed that the results of the course functioned widely in many subjects, in many situations; it also showed that the students found the non-mathematical material more interesting and believed it to be more useful.

APPLICATIONS

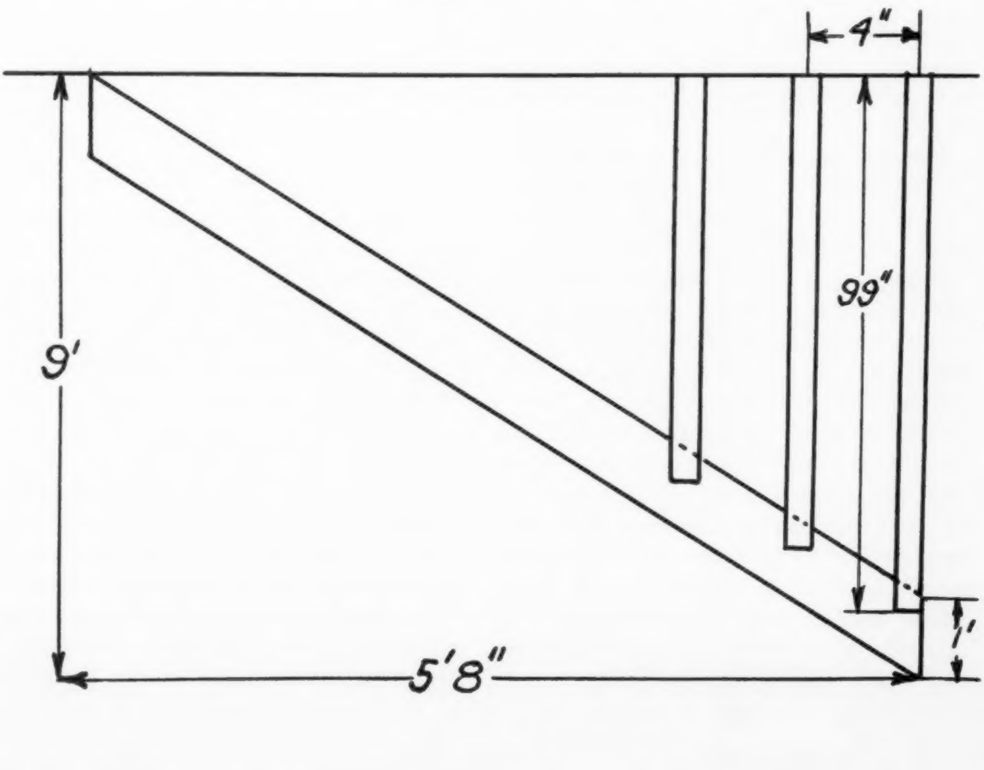
Edited by Sheldon S. Myers
University School, Ohio State University, Columbus, Ohio

Since this column was written before the first fall issue, the reader would not have had a chance to respond to our request for contributions. We hope to hear from many contributors to future columns. Last summer, a colleague of ours, Mr. Lyman Peck of the Ohio State University mathematics department, hearing of this department beforehand, submitted the following very practical and appropriate application of arithmetic series.

C.A1. 1 Gr. 10–12 Arithmetic Series in the Home

An open cellar steps against the basement wall of his home proved hazardous for Mr. Peck's little daughter. He made plans to nail wooden slats to the open side as shown in the drawing. In order to purchase slats, he found that he needed to know their total length, which he computed very conveniently by arithmetic series.

The two basic formulas needed are: $L=a-(n-1)d$ and $S=n(a+L/2)$, where L = last term, a = first term, n = number of terms, d = common difference, and S = sum of the series. a, n, and d can be computed directly from the data in the drawing. The solution will be given next month.



One year while directing a testing program for a high school, the writer came across the following interesting application of signs, parentheses, and brackets in elementary algebra.

Al. 3 Gr. 9–10 Developing a Formula for Grading a Standardized Test

Many standardized tests have their scores computed by subtracting the wrongs from the rights. Allowing S=score, R=rights, W=wrongs, 0=omitted, show that in a test of 50 items $S=2R+0-50$. (Solution next month.)

Now that the crickets are slowing up for the season, it may be possible to capitalize on this fact in elementary algebra when students are using simple formulas.

Al. 4 Gr. 9–10 Vocal Thermometers

Bert Holmes, the naturalist, has discovered that the rate of cricket chirping is a linear function of the temperature. Consequently, by statistical means, he converted experimental data on cricket chirping and temperature to the following equation involving these two variables:

$$C=3.777T-137.22$$

Where C=the chirps per minute
and T=corresponding Fahrenheit temperature

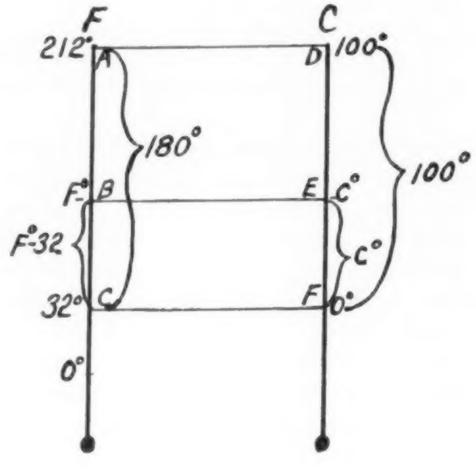
This can be made explicit in terms of T:

$$T=\frac{1}{3.777}C+\frac{137.22}{3.777}$$

Armed with such formulas, students might be moved to test them out in practice. A reliability of .9919 is claimed for them. Here are some references: *The Scientific Monthly*, Vol. XXV, Sept. 1927, pp. 261–264 and *Applied General Statistics* by Croxton and Crowden. For those of you who teach the slope-intercept form ($Y=mX+b$) in the first or second year, these two formulas would make interesting graphs to study in relation to this form. These graphs would show vividly the existence of a "critical temperature" below which chirping does not occur.

P.G. 1 Gr. 10–11 Using Parallel Lines to Derive the Fahrenheit-Centigrade Relation

Using the Fahrenheit and Centigrade thermometers as transversals, the following parallel lines can be drawn:



By a well-known geometry theorem, the following proportion can be established:

$$\frac{BC}{AC}=\frac{EF}{DF}.$$

From the figure, $BC=F-32$, $AC=180$, $EF=C$, $DF=100$. By substituting in the above proportion, we get:

$$\frac{F-32}{180}=\frac{C}{100}.$$

Solving for F: $F=9/5\ C+32$.

(For an experimental way of deriving the same relationship by slope-intercept, see *School Science and Mathematics*, November 1948, pp. 645–646.)

Al. 5 Gr. 9–10 Graphs on Anti-Freeze Mixtures in Car Radiators.

Before you can find the number of quarts of anti-freeze needed to safeguard your car radiator to a certain temperature, you must find the *capacity* of the radiators. Following are some of these capacities

which can be found on charts at filling stations: Ford 6—16 quarts, Chevrolet 6—16 quarts, Cadillac—18 quarts, Pontiac—18 quarts, Frazer—14 quarts, Studebaker Champion—10 quarts, Crosley—4 quarts, Lincoln R-9—35 quarts, Buick 6—14 quarts, Mercury—22 quarts, and Chrysler New Yorker—22 quarts. Add one quart to each of these for a hot water heater.

When anti-freeze is added to your radiator, some of the water must first be drained off. As more and more anti-freeze is added, the freezing point of the mixture is lower and lower. Different anti-freezes, such as Sohio Permaguard, ethyl alcohol, and glycerine, do not lower the freezing point at the same rate. However, in all cases the *rate* of lowering is nearly constant, so that the graph of the quarts of anti-freeze and the freezing point is practically a straight line. Following are tables showing how Permaguard safeguards 15, 18, and 35 quart radiators.

15 quarts		18 quarts		35 quarts	
qts.	T°F	qts.	T°F	qts.	T°F
3	17.5	3	21	6	19.5
4	10	4	13	7	16
5	2	5	6	8	13
6	− 8.5	6	− 1	9	9
7	−20.5	7	−10	10	5

After plotting three graphs for each of the above tables, students will have direct experience with approximate linear functions as they occur in science. With the information supplied in this application, they will be able to answer questions like the following: How much Permaguard is required to safeguard a Frazer (with heater) to 2°, −5°, 6°, and −25°? The meaning of interpolation and extrapolation can be taught in this setting as well as dependent and independent variables. Another question might be: To what temperature will $3\frac{1}{2}$ quarts safeguard it?

Here is a wintry, junior high problem saved for this December issue.

Ar. 4 Gr. 7–9 Snow on the Roof

After the first snow, your students will be surprised at the weight of the snow deposited on the roof of their house and on the roof of their school building. In Central Ohio, builders are required to contruct roofs that will withstand 40 pounds per square foot. In other sections of the country where more snow falls, this required figure may be higher. Suppose that 3.1 inches of snow fell on a roof 30 feet by 40 feet. Weather Bureau experts will tell you that 3.1 inches of snow will melt to .31 inches of water which weighs 1.56 pounds when spread over a square foot of surface. Thus the above roof will have $30\times 40\times 1.56$, or 1872 pounds of load on it. This lacks by 128 pounds of being a ton. A public building 110 feet by 120 feet would have over 10 tons of load after the above snowfall. Since snow piles up ten times what it would be if melted to water, your students can figure snow loads on buildings and practice ratios at the same time. The Weather Bureau figures allow 5.03 pounds for every inch of depth of water spread over a square foot. Your students could compute how many inches of snow must fall to exceed the load limit in your area.

Here is the solution to last month's problem:

Al. 2 Gr. 10–12 Dimensional Relationships in Physics

1. $20.4 \div 3.4 = 6$ cubic eels
2. $3.4\times 50 = 170$ groggins
3. $35\times 3.4\times 49.5 = 5890.5$ caliacas
4. $\dfrac{85\times 3.4}{26.3} = 10.9$ quatinas
5. $\dfrac{26.3\times 3.5}{3.4} = 27.07$ cubic eels
6. $\dfrac{990}{49.5} = 2$ groggins

$2.0 \div 3.4 = .58$ cubic eels.

MATHEMATICAL RECREATIONS

Edited by Aaron Bakst

School of Education, New York University New York 3, N. Y.

During the tenth Summer Meeting of the National Council of Teachers of Mathematics several hundred members of the Council and their guests attended a Terrace Supper. Their host was the Milwaukee County Suburban Mathematics Teachers Club. Among the many surprises and entertainment that were showered on the assembled diners was a test (or quiz). We shall pass with silence the statistical report on the performance. However, this test kept everybody in a dither, that is, until Miss Margaret Joseph graciously supplied the answers.

This department, represented by its editor, failed to obtain a perfect score. But, now, being in a position to supply the members of the National Council with the correct answers, this department, secured Miss Joseph's permission to publish the test in its entirety. If you care to try your skill at it, or if you wish to try it out with your classes in geometry, you will find the answers to this test in a forthcoming issue of The Mathematics Teacher.

The words, which should be filled in the blank spaces, are all within the vocabulary-scope of tenth-year mathematics. Only one word should be placed in any given single blank space.

A Mathematical Romance

______ ______ who had served in the war against the ______ ______, took advantage to the G. I. Bill and enrolled at the U of W to work for his Master's ______ He loved the beautiful campus but the ______ of the walk to Bascom Hall was so great that it made him walk ______ This annoyed him for he had been trained to walk ______. When classes were over he often went to Picnic ______ to relax and ______ the ______ about him.

One day as he sat at the ______ of a large pine tree, a coed stopped and picked up a ______ that had fallen from the tree. She admired its ______ of form. She smiled so he knew that that was a ______ that she was ______ to be friendly. He learned that her name was ______ and she was ______ ______. Her father was ______ ______. The Major was not a handsome man; in fact ______ called him ______ but he had a good ______, and he dressed in the ______ of fashion. Their tastes and interests were ______ and soon they became ______ pals.

He called upon her on ______ evenings and took her to several campus ______. He composed several ______ verses about her as he played on her piano. Some days he seemed moody and went off on a ______. At such times he expressed interest in the ______ views of a group of ______ students. She thought nothing ______ about it when he ______ abruptly at the end of the semester for he said that he would ______ to her.

She thought it would be very romantic to ______ with him when he was at a ______. She felt certain that this would result in a ______ and that they would be married. Perhaps it should have been called a ______ for she did not think it required proof. She even dreamed of an ______ honeymoon. Her father had ______ that they would live in the house he planned to ______ on the ______ ______ to his, for there was ample ______ on the fashionable ______ where they lived.

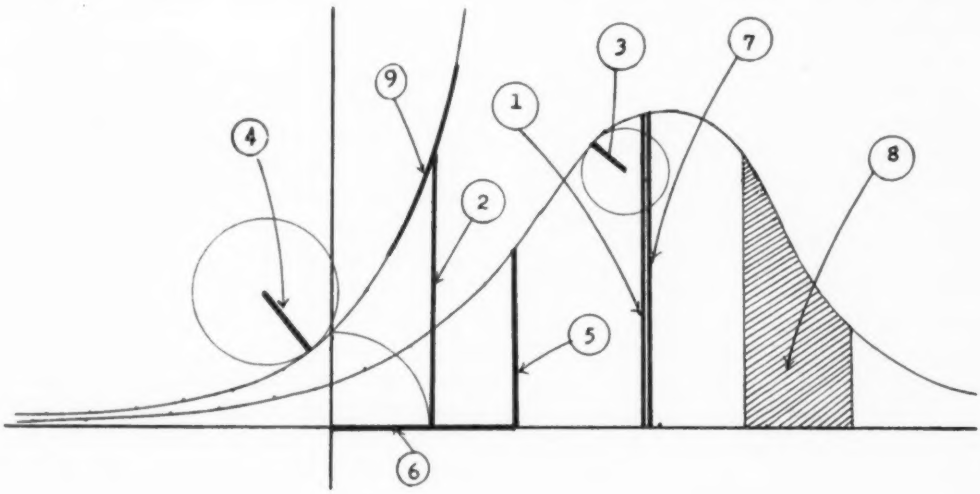
A considerable ______ elapsed and no letter came from the Major. Her suffering was ______ and she would ______ with no one. She even refused to eat her favorite ______. She just sat staring into ______ and every now and then she heaved a ______.

One day their maid brought a telegram on which her name was ______. It was from the Major stating that he would not return because his mother told him not to be involved in a ______. She went ______ to her room. She maintained a brave ______ until she was alone. Then everything seemed to ______ about her and she fell in a swoon. The family seemed relieved that he was out of her social ______, for they knew that he was just a false ______. So she tossed back her head and promised never to be ______ again.

Here is the promised solution of the Christmas Greeting which appeared in the November issue of THE MATHEMATICS TEACHER. It should be noted that each part of the drawing which is represented by a thickly drawn line may be represented by a mathematical symbol. These are identified by numbers. Thus we have: (1) M, (2) E, (3) R, (4) R, (5) Y, (6) X, (7) M, (8) A, and (9) S.

The two letters M are associated with the mean and median (or mode) of the normal distribution curve. The exponential curve could have probably offered some difficulties in identification. However, some hints were offered by the arc of the circle drawn with the origin as a center. Thus the distance from the origin to the exponential curve is 1; the equation of the curve is $y=e^x$, and when $x=0$, $y=1$; and when $x=1$, $y=e$.

Perhaps some of the readers will design another Christmas Greeting. This department will gladly publish other original designs. It would be interesting to have an Easter Greeting in mathematical form.



MATHEMATICAL MISCELLANEA

Edited by PHILLIP S. JONES
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11. *Mental Substitution as a Method of Solving Equations*

Probably every teacher of mathematics discovers sooner or later that it is desirable to be able to explain problems and justify processes in more than one way as a student who is puzzled by one method of solution may find another quite easy to follow.

In the study of equations, for example, the method of solving by mental substitution may not only be used to introduce the topic, but also may well be extended to more difficult types of equations, at least for those students who experience difficulty in using other methods.

The seventh or eighth grade student solves one step equations such as $x+3=7$, $2m=8$, or $n/3=2$ by *mental* substitution. That is, he thinks in solving the first equation: "4 plus 3 equals 7, therefore x equals 4."

Left to his own devices, the better student in this age group will solve equa-

tions of the types, $3x+2=8$, $5y-1=14$, $x/3+2=17$, and $n/2-5=10$ by the same method. His thought process in solving the first equation is about as follows: "$3x$ plus 2 equals 8. Since 6 plus 2 equals 8, $3x$ must be equal to 6, and x equals 2." Or, in the last equation above: "$n/2$ minus 5 equals 10, $n/2$ must be equal to 15. 30/2 equals 15, so n must be equal to 30."

He will probably not be able to record his thought process in writing, and will therefore write just the original equation, $n/2-5=10$, and the solution, $n=30$, omitting the intermediate steps.

This suggests a desirable transitional step between solution by this mental substitution and solution by more advanced methods. The student may continue to use his normal thought process of mental substitution while learning to write the results of his thinking in the following manner:

$$\frac{n}{2}-5=10$$

$$\frac{n}{2}=15$$

$$n=30$$

He should then be asked to study the first two equations and tell how the second is different from the first. He will probably say, "5 has been added to 10." His attention should then be called to the fact that 5 has also been added to the left hand member, which was 5 less than $n/2$ and is now just $n/2$. In other words, while solving by substitution, he has also, if unknowingly, used the method of adding 5 to each side of the equation. The second and third equations should be studied and compared in the same way.

Certain students will feel more confident in using this method than in using any other, at least for a time, and in solving equations of the type shown above.

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12. *Big Numbers*

Big numbers have had a fascination in all ages for both men and children. Archimedes' *Sand Reckoner*, the religious lore of the Hindus,[3] and the "googol" of Kasner,[4] which was once a popular news item, all bear witness to this.

A recent flurry of activity in relation to big numbers has included serious and pastime work directed toward several ends.

R. C. Archibald recently called attention to a new advance in computing the answer to the old question of what is the largest number which can be expressed with three digits.[5] The answer is 9^{9^9}. In an earlier article[6] Archibald had noted that C. A. Laisant had shown in 1906 that this number would have 369,693,100 digits if expressed in decimal notation, and that if the number were printed with one-fifth inch figures it would take a strip of paper 1166 miles, 1690 yards, 1 foot and 8 inches long.

In 1913 A. C. D. Crommelin found the first 28 and last 3 digits of this number and went on to remark, "Knowledge of 30 figures out of 300 million may seem trifling, but in reality the error involved . . . is only one part in a thousand quadrillion." (A quadrillion in England means 10^{24} rather than 10^{15} as in this country.)

The recent extension was the publication of the first 60 and last 24 digits of this number by Chr. Weiss in 1941.

Archibald's note spurred H. S. Uhler to compute and publish in 1947 log 9^{9^9} to 250 decimal places.[7] Uhler did this as

[3] See Sir Edwin Arnold, *The Light of Asia*, and L. C. Karpinski, *The History of Arithmetic* (Chicago: Rand McNally and Co., 1925), pp. 38 ff.

[4] Edward Kasner and James Newman, *Mathematics and the Imagination* (New York: Simon and Schuster, 1940), pp. 18 ff.

[5] R. C. Archibald, "A Huge Number," *Mathematical Tables and Other Aids to Computation*, II (1946), pp. 93–94.

[6] R. C. Archibald, "Huge Numbers," *American Mathematical Monthly*, XXIII(1921), pp. 393–394.

[7] Horace S. Uhler, "Huge Numbers," *Mathematical Tables and Other Aids to Computation*. II (1947), pp. 224–225.

recreation and relief from his determination of the factorability of Mersenne numbers.

Mersenne numbers, M_n, are numbers of the form 2^n-1 where n is a positive integer. Prime Mersenne numbers are of especial interest because a theorem going back to Euclid states that if 2^n-1 is prime then $2^{n-1}(2^n-1)$ is a "perfect number." Perfect numbers are numbers which are equal to the sum of all their divisors other than themselves. For example

$$6=1+2+3, \qquad 28=1+2+4+7+14.$$

These are derived from M_2 and M_3 and are given by $2^1(2^2-1)$ and $2^2(2^3-1)$ respectively. There are only 12 known Mersenne primes. The last to be discovered was M_{107} found to be prime in 1914. Recent progress began in 1944 when Uhler showed M_{157} to be composite. He has since then shown the other five previously untested Mersenne numbers having $n \leqq 257$ to all be composite.

The largest number of any kind which is actually known to be a prime is then

$$M_{127}=170,\ 141,\ 183,\ 460,\ 469,\ 231,\ 731,\ 687,\ 303,\ 715,\ 884,\ 105,\ 727.$$

This is generally supposed to have been shown to be prime by Lucas in 1876. Aimé Ferrier of Ellier, France, questions this and gives credit to Fauquemberg in 1914.[8]

Ferrier himself has reported in the last year that $2^{61}-1$ and $2^{61}+15$ are both prime while the integers between them are composite. Hence these two numbers are the largest known pair of consecutive prime numbers.[9]

13. Fractionals and the Unit Point

Many of the modern electronic digital computers use the binary system. There is continuing publicity and advocacy for the general use of a duodecimal system. Some teachers have found the discussion of scales of notation valuable at several grade levels to increase both students' interest in and understanding of arithmetic processes. All of these factors have called attention to the fact that there are several terms in current mathematical usage that are somewhat more limited in their application than they should be. Clear thinking would be facilitated by the use of terms that are semantically more proper.

The origin of the use of decimals and the decimal point is generally attributed to Simon Stevin, dating from the publication of his *De Thiende* at Leyden in 1585. At that time it was not generally recognized that numbers could be systemized on other than the ten-base. For this reason these terms seemed quite proper in their construction and application. But, when it became known that the binary and other bases offered advantages in specific applications, these terms had to be limited to their use or be confusingly inaccurate. The use of the terms "decimal" and "decimal point" is proper only with the ten-base, and is definitely improper with other bases and for any general statement.

The decimal point is sometimes called the separatrix. This terminology is quite correct although the word is itself somewhat ponderous. H. K. Humphrey of Winnetka, Illinois, has suggested the term "unit point," which is simple, clear, and nicely correlative with its predecessor.

Similarly, the application of the term "decimals" to systemic fractions is correct only for the ten-base, and the use of the preferable term "fractionals" is suggested as being free from the connotation of a specific base.

The form of these terms is simple and unstilted. While they may initially seem strange and may require a second thought for selection, in a little while they become familiar and easy to use. They avoid improper implications, and aid in clarifying thought.

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[8] Aimé Ferrier, "Note on the Factors of 2^n+1," *Mathematical Tables and Other Aids to Computation*, III (1949), pp. 496–497.

[9] *Op. cit.* IV (1950), pp. 124–125.

14. Introducing the Multiplication Table for Signed Numbers

The following useful device for teaching multiplication of signed numbers was shown me recently by Professor Walter J. Bruns of Syracuse University. An application of Hankel's principle of permanence,[1] it introduces the postulational approach to number in a simple, elementary way.

In the familiar multiplication table for positive integers and zero, when one of the factors is decreased by one, the product is decreased by the other factor. For instance, in the 3-table at the right below, the product is diminished by 3 as we decrease the multiplier by one and move up the 3-table column (and also as we move to the left in the 3-table row).

$0\times0=0$	$1\times0=0$	$2\times0=0$	$3\times0=0$
$0\times1=0$	$1\times1=1$	$2\times1=2$	$3\times1=3$
$0\times2=0$	$1\times2=2$	$2\times2=4$	$3\times2=6$
$0\times3=0$	$1\times3=3$	$2\times3=6$	$3\times3=9$

Why stop at zero now that our interest in integers no longer stops there? Let us postulate that this method of building up the table continues to apply when one (or both) of the factors is a negative integer. This is an adventurous experiment and may get us into trouble but, at least, we can investigate and see what will happen. If we find that it "works" (that is, if the commutative, associative, and distributative laws hold) we shall find the extended table as useful in all its parts as we have previously found its lower right hand part. The basic laws need not be known to the students by name but will, to some extent at least, have become familiar in principle and important in practice.

Let us now proceed to extend the table according to the postulated plan:

In the 2-table, as the multiplier is decreased from zero to -1, the product will drop to -2, and, as we move further up, lowering the multiplier by one, the product is diminished by 2 each time, to -4, -6, -8, etc. Similarly, in the 3-table in the bottom row, as we lower the first factor from zero to -1, the product is diminished by the other factor (in this case by 3) and becomes, successively, -3, -6, -9, etc.

Turning now to the top row where the "minus five" table is being formed, we are to obtain each product from its right hand neighbor by subtracting minus five each time. But subtracting minus five gives the same result as adding plus five. Therefore, in this row, as in all the rows above the zero row, the products step up as we go to the left and we notice some startling results to the left of the zero column. These same results are also obtained as we move up the columns where the (-1) table, the (-2) table, the (-3) table, etc, have now been formed. As we keep on moving upward and to the left, filling in rows and columns, we obtain a novel, understandable, and unforgettable array.

The validity of the commutative law is brought out by the interchangeability of rows and columns. The other laws can be

[1] *Editor's Note:* The "Princip der Permanenz der formaten Gesetze" which appeared in H. Hankel, ***Theorie der Complexen Zahlensystem***, Leipzig: 1867, like Poncelet's principle of continuity in geometry, is fruitful in suggesting new extensions of concepts and in helping one to perceive interrelationships within mathematics already developed. Both principles lack rigor and should be used with care. For example the application of the former principle to division would through the sequence $8\div4=2$, $8\div2=4$, $8\div1=8$, $8\div0=?$, $8\div(-1)=?$ suggest that $8\div(-1)$ should be greater than $8\div0$ and $8\div1$.

$(-2)\times(-5)=10$	$(-1)\times(-5)=5$	$0\times(-5)=0$	$1\times(-5)=-5$	$2\times(-5)=-10$
$(-2)\times(-4)=8$	$(-1)\times(-4)=4$	$0\times(-4)=0$	$1\times(-4)=-4$	$2\times(-4)=-8$
$(-2)\times(-3)=6$	$(-1)\times(-3)=3$	$0\times(-3)=0$	$1\times(-3)=-3$	$2\times(-3)=-6$
$(-2)\times(-2)=4$	$(-1)\times(-2)=2$	$0\times(-2)=0$	$1\times(-2)=-2$	$2\times(-2)=-4$
$(-2)\times(-1)=2$	$(-1)\times(-1)=1$	$0\times(-1)=0$	$1\times(-1)=-1$	$2\times(-1)=-2$
$(-2)\times0=0$	$(-1)\times0=0$	$0\times0=0$	$1\times0=0$	$2\times0=0$
$(-2)\times1=-2$	$(-1)\times1=-1$	$0\times1=0$	$1\times1=1$	$2\times1=2$
$(-2)\times2=-4$	$(-1)\times2=-2$	$0\times2=0$	$1\times2=2$	$2\times2=4$
$(-2)\times3=-6$	$(-1)\times3=-3$	$0\times3=0$	$1\times3=3$	$2\times3=6$

tested in random numerical cases. When the rules of sign made manifest in the table are postulated and tested for positive and negative fractions, much timely computational practical will result.

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15. A Method for Speedy Factoring of the General Quadratic Trinomial

The following discussion presents a method for obtaining, more rapidly and more easily, the binomial factors $(ax+b)$ and $(cx+d)$ of the general quadratic trinomial $acx^2+(ad+bc)x+bd$ than is possible by a systematic blind combining of all possible factors.

Luck, trial and error, and groping, present in the usual methods, are virtually eliminated.

In the following discussion it is assumed that:

1. The three terms of the trinomial are arranged in ascending or descending powers of some letter.
2. a, b, c, and d are integers.
3. Any common monomial factors have been removed.

The process will be considered in two cases: Case I, used when the numerical coefficient $(ad+bc)$ of the middle term is *odd*, Case II, used when the numerical coefficient of the middle term is *even*.

CASE I

When the numerical coefficient $(ad+bc)$ of the middle term is odd, it must be the sum of one even product and one odd product. The odd product will be assumed to be ad and the even product bc. The odd product ad must be obtained from two odd factors, one being an odd factor of the numerical coefficient of the first term and the other being an odd factor of the numerical coefficient of the third term.

Thus, for the trinomial to be factorable the numerical coefficients of the first and third terms must both contain at least one odd factor, and, further, not all four factors of the first and third coefficients can be odd because the middle coefficient would then be an even number, the sum of two odd products. That is, when all three coefficients of the general quadratic trinomial are odd, it is not factorable.

To illustrate the above, consider the factoring of the trinomial $6x^2+17x+12$. The middle coefficient 17 is odd and must be the algebraic sum of an odd product and an even product. The odd product results from an odd factor of 6 and an odd factor of 12. Thus the factors of 6 must be (3) (2) and the factors of 12 must be (4) (3) and since the 3's cannot occur in the same binomial and likewise 2 and 4 cannot occur in the same binomial (there would have been a common monomial factor in the trinomial) the factors of the trinomial must be $(3x+2)(4x+3)$. There is no other possibility for factoring this trinomial if the trivial factors of 6 and 12, i.e. (6) (1) and (12) (1), with the resulting obvious middle coefficient of $+73$ are excluded.

For a second example, if the middle coefficients of the following are *odd*,

$$24x^2 \pm ?x \pm 48$$

the only possible setup for the factors is

$$(3x \pm 16)(8x \pm 3)$$

if the trivial factors of 24 and 48, i.e., $(24)(1)$ and $(48)(1)$ are excluded by the absence of the obvious middle coefficients of ± 1151 and ± 1153.

Again in $56-11x+12x^2$, the factors of 56 must be $(7)(8)$ and the factors of 12 must be $(4)(3)$. 4 and 8 cannot occur in the same binomial, thus the only factoring possibility is $(8+3x)(7-4x)$ if the trivial factors are excluded.

For a test, what are the pairs of factors of 28 and 40 in the trinomial $28x^2-3x-40$? There is only one possibility when the middle numerical coefficient is odd, (excluding the trivial factors with the resulting middle coefficient of 1121).

To factor the more difficult $72x^2+41x-45$, an odd factor must be obtained from

72. There are two: *3* and *9; 3* cannot be used as this would force *24* (with its factor of *3*) into the other binomial. Then there would be *3's* in both binomials and one of the binomials must take a factor of 45 that would contain a *3*. Thus *9* must be used as a factor of *72* and the only possibility is $(9x-5)(8x+9)$.

However, the rare exception, where there is more than one possibility, does exist for this case. This occurs only when the first or the third coefficient has two or more odd prime factors. An example of this exception happens in factoring the trinomial, $20x^2-49x+30$, since *30* has two odd prime factors (*3* and *5*). The factors of the trinomial are $(5x-6)(4x-5)$ but the second possibility of $(5x\pm 2)(4x\pm 15)$ must have required consideration.

CASE II

When the numerical coefficient $(ad+bc)$ of the middle term of the trinomial is *even,* it must be the sum of two even products or the sum of two odd products. (It is amazing that, in this case, both factors of the first coefficient must be odd or both factors must be even, and also both factors of the third coefficient must be odd or both must be even).

To illustrate the above, in factoring $56x^2+50x-25$ the factors of *56* must be $(14)(4)$ or $(28)(2)$; the factors of *25* must be $(5)(5)$, excluding the trivial ones.

To summarize both cases:

CASE I. When the middle coefficient is *odd,* an *odd* factor must be used from an even first or third coefficient.

CASE II. When the middle coefficient is *even,* two *even* factors must be used from an even first or third coefficient. (In this case, not all original coefficients can be even as it is assumed that there is no common monomial factor).

The treatment of Case I has been more extensive than that of Case II since it is the opinion of the writer that the more rigorous the text the greater the frequency of occurrence of the odd middle coefficient.

To factor a trinomial with coefficients that are fractional, one, obviously, can change the three terms of the trinomial to equivalent terms having a common denominator, remove the common monomial denominator factor, and proceed as in the above.

For the "doodler" some interesting problems are:

Can $15x^2\pm ?x+15$ be factored if the middle coefficient is odd and there is no common monomial factor?

When factoring the trinomial $18x^2\pm ?x+54$, what is the only possibility for the missing coefficient which will lead to unique, prime, binomial factors?

(Middle term odd and the trivial cases excluded.)

Factor: $14-(11x/4)+3x^2$.

The writer has used this method as a follow-up to the usual textbook routine in average classes. This follow-up has brought a rewarding glow of appreciation to the faces of the students.

The main value of the method comes from the relationships with odd and even numbers that are observed and used in the method.

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"Special Delivery" From Our Mailbox

"I would like to make a suggestion for a page in the magazine. We hear so much these days about exchange teachers and all realize the fine opportunity this gives for understanding and growth. Why not have a page devoted to publications of names and positions of people interested in exchange? These might be both national and perhaps international. Those interested in exchange could give their own location and list places where they would like to go for exchange. In this way people interested would be brought together."

Frances Lethlean, Public Schools, Glencoe, Illinois

What do you think? Send us your **X = POP** (Poll on Postcard). Editor.

TOPICS OF INTEREST TO MATHEMATICS TEACHERS

Edited by William L. Schaaf
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Mathematics and Art

Claude Bragdon once observed that to some people, any juxtaposition of the idea of mathematics and the idea of art would seem far fetched. Devotees of modern non-objective painting apparently feel the same way. To be sure, artistic creations do not invariably stem from mathematical concepts, nor does the execution of works of art depend solely upon mathematical considerations. But basically, pure mathematics and creative art share certain common characteristics: both admit of unlimited use of the imagination; both are creations for the sake of creation; both are pure invention; both reveal abstract orderliness imposed by the human mind. And when the artist portrays reality, he takes into account the mathematical aspects of real objects, be they the works of man, or found in Nature.

Two fairly recent publications are of considerable interest: Matila Ghyka's *The Geometry of Art and Life*, and William Ivin's *Art and Geometry*. Perusal of these two delightful little books should dispel any doubt that art and mathematics are intimately related. The following guide to source materials may serve as an introduction. Unfortunately, limitations of space preclude annotations: in many instances, however, the title of the book or article sufficiently indicates its contents. This department invites your suggestions; it is our purpose to present material of optimum value to our readers.

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2. ORNAMENT AND DESIGN; PATTERN; GEOMETRIC DRAWING

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Baravalle, H. von. *Geometry at the Junior High High School Grades and the Waldorf School Plan*. Garden City, N. Y., Adelphi College, 1948, 32 p.

Bradley, A. Day. *The Geometry of Repeating Design and Geometry of Design for High Schools*. New York, Teachers College, Columbia University, 1933. 131 p.

Carroll, John. *Pattern Drawing and Design; the Application of Geometrical Drawing to the Construction of Ornament and the Planning of Patterns*. London, Burns and Oates, Ltd., 1902. 112 p.

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3. DYNAMIC SYMMETRY

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According to the Principles of Proportion Discovered by Jay Hambridge. Boston Museum of Fine Arts, 1922. 235 p.

Hambridge, Jay. *Dynamic Symmetry: The Greek Vase*. New Haven, Yale University Press, 1920. 161 p.

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4. The Golden Section

(a) Books

Baravalle, H. von. *Die Geometrie des Pentagrammes und der Goldene Schnitt*. Stuttgart, Walforf Verlag, 1932.

Engelhardt, Rudolph. *Der Goldene Schnitt im Buchgewerbe*. Leipzig, J. Maser, 1922. 282 p.

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(b) Articles

Baravalle, H. von. "The Geometry of the Pentagon and the Golden Section." The Mathematics Teacher, 1948, 41: 22–31.

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Gaudet, C. "Golden Measure and Greek Art." *Spectator*, 1920, 124: 235–236.

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BOOK SECTION

Edited by J. Stipanowich
Western State College, Macomb, Illinois

This section presents the latest books which have been received for review in The Mathematics Teacher. Reviews of many of these books will appear in the monthly issues. Members of the Council are invited to send us further comments or corrections of errors relating to any of the books mentioned. In addition, a free loan service has been arranged whereby any member may borrow any of the books listed for a period not to exceed two weeks. Requests should be addressed to The Mathematics Teacher, 212 Lunt Building, Northwestern University, Evanston, Illinois.

BOOKS RECEIVED

Elementary School

Arithmetic 5, The World of Numbers, by Dale Carpenter, Supervisor of Mathematics, Los Angeles City Schools; Edith M. Sauer, Principal, Lincoln and Jefferson Elementary Schools, Springfield, Massachusetts; and G. Lester Anderson, Dean of Teacher Education, New York City Colleges. Cloth, 316 pages, 1950. The Macmillan Company, 60 Fifth Avenue, New York, $1.68.

Arithmetic 6, The World of Numbers, by Dale Carpenter, Supervisor of Mathematics, Los Angeles City Schools; and Dorothy Leavitt Pepper, Elementary Teacher, Los Angeles City Schools. Cloth, 316 pages, 1950. The Macmillan Company, 60 Fifth Avenue, New York. $1.68.

High School

General Mathematics

Zahl und Raum, I Teil, Geometrie, by Fritz Malsch, Ober-Studiendirektor in Idstein i. Ts. Paper, 127 pages, 1949. Verlag Quelle und Meyer G.M.B.H., Heidelberg. Lehrmittel-Verlag G.M.B.H., Offenburg (Baden).

Zahl und Raum, II Teil, Arithmetik und Algebra, by Fritz Malsch. Paper, 188 pages, 1949.

Zahl und Raum, III Teil, Arithmetik, Algebra, Analysis, Stereometrie, by Fritz Malsch. Paper, 126 pages, 1949.

Zahl und Raum, IV Teil, Trigonometrie, Analytische Geometrie, by Fritz Malsch. Paper, 110 pages, 1949.

Plane Geometry

Geometric Constructions, by Aaron Bakst, School of Education, New York University. Paper, 59 pages, 1950. New York University Bookstore, Washington Square, New York. $0.50.

Dynamic Plane Geometry, by David Skolnik, Central Commercial and Technical High School, Newark, New Jersey; with the editorial assistance of Miles C. Hartley, Assistant Professor of Mathematics, Chicago Undergraduate Division, University of Illinois. Cloth, x+288 pages, 1950. D. Van Nostrand Company, Inc., 250 Fourth Avenue, New York. $2.56.

College

1st Year Mathematics

Basic Mathematical Analysis, For Junior and Senior Colleges, by H. Glenn Ayre, Professor of Mathematics, Western Illinois State College, Macomb. Cloth, xvi+584 pages, 1950. McGraw-Hill Book Company, Inc., 330 West 42nd Street, New York. $5.00.

Business Mathematics

Business Mathematics, Third Edition, by Cleon C. Richtmeyer, Central Michigan College of Education; and Judson W. Foust, Central Michigan College of Education. Cloth, xvii+441 pages, 1950. McGraw-Hill Book Company, Inc., 330 West 42nd Street, New York. $3.50.

Statistics

Contributions to Mathematical Statistics, by R. A. Fisher, Department of Genetics, University of Cambridge. Cloth, 656 pages, 1950. John Wiley and Sons, Inc., 440 Fourth Avenue, New York. $7.50.

An Introduction to the Theory of Statistics, 14th Edition, by G. Udny Yule, formerly Reader in Statistics, University of Cambridge; and M. G. Kendall, Professor of Statistics, University of London. Cloth, xxiv+701 pages, 1950. Hafner Publishing Company, Inc., 31 E. 10th Street, New York. $7.00.

Advanced Mathematics

George David Birkhoff, Collected Mathematical Papers, 3 Volumes. Cloth, lvii+754, 983, and 897 pages respectively, 1950. American Mathematical Society, 531 West 116th Street, New York. $18.00.

Table of the Bessel Functions $Y_0(z)$ and $Y_1(z)$ for Complex Arguments, by the Computation Laboratory, National Bureau of Standards. Cloth, 427 pages, 1950. Columbia University Press, 2960 Broadway, New York. $7.50.

The Location of Critical Points of Analytic and Harmonic Functions, Colloquium Publications, Vol. 34, by J. L. Walsh, Harvard University. Cloth, viii+384 pages, 1950. The American Mathematical Society, 531 West 116th Street, New York. $6.00.

The Theory of Valuations, Mathematical Surveys, No. 4, by O. F. G. Schilling. Cloth, vii+253 pages, 1950. The American Mathematical Society, 531 West 116th Street, New York. $6.00.

REVIEWS

Second Year Algebra: New Edition, **by Raleigh Schorling, Rolland R. Smith, and John R. Clark. Cloth, xii+500 pp., World Book Co., 1950. $2.20.**

This book is intended to supplement a second-year course in algebra. Such a course, the authors believe, is to train both "those students who plan to engage in scientific, professional, or skilled professional work" and those students who have proved capable of doing a good job in a first course in algebra and enjoyed the subject.

The first two of the nineteen chapters are devoted to a review of the fundamental operations and linear equations. These two chapters contain nine inventory tests, several other objective-type tests, and a section on problem solving and problem analysis.

The remaining chapters deal with formulas, special products of binomials, fractions, the coordinate system, linear and quadratic equations, exponents, logarithms, simple series, ratio and variation, and numerical trigonometry. In addition to these is an optional chapter, "Rates of Change," and a fifteen page section on "Applications of Conic Sections," with numerous illustrations.

Ability levels are provided for by grading exercises according to difficulty and by the inclusion of twenty-nine sections on optional topics, such as, compound interest, the slide rule, the point slope equation and the distance formula. Throughout the book are placed cumulative reviews, reviews of geometry, and several historical sketches. Each chapter is summarized by two tests.—R. W. MOLLENDORF, McKinley High School, Chicago, Illinois.

Plane Trigonometry by John J. Corliss and Winifred V. Berglund. Cloth, xii+388 pp. Houghton Mifflin Company, 1950. $3.00.

The authors state that the student's mathematical development should be founded on reasoning powers rather than on memory. The student is asked to derive formulas—not memorize them. Definitions, of course, are to be committed to memory. The text brings out clearly the dual role of trigonometry as a course designed to exercise the power of analysis as well as a course in computaion.

A brief discussion of the function concept precedes the definitions of the trigonometric functions. These definitions are later specialized for the acute angle. The material is arranged so that important topics such as radian measure, trigonometric equations, identities and inverse functions appear more than once. The chapter on applications includes problems taken from mechanics, physics, and air navigation. Complex numbers are well treated with twenty-five pages devoted to this important topic. No mention is made of the use of the slide rule. Answers are provided for about half of the problems.

The rules for finding the characteristic of a common logarithm of a number may lead to confusion on the part of the student inasmuch as the rules begin: "The characteristic of a number. . . ." Furthermore, no distinction is made between formula and identity.

On the whole, however, the book is reasonably complete and easily worth considering as a textbook. It would seem better suited for college than for high school.—ELBERT C. HUBBARD, Parks College of Aeronautical Technology, St. Louis University, East St. Louis, Illinois.

Geometry: Meaning and Mastery by Samuel Welkowitz, Harry Sitomer, and Daniel W. Snader. Cloth, v+506 pp. The John C. Winston Co., 1950. $2.60.

The objectives and methods of this text are indicated by the titles of two of its early chapters—"The Size and Shape of Things" and "Experimentation and Deduction." The inclusion of experimental proofs for many theorems, a great many illustrations of simple applications of geometry, and extended sections on the forms of reasoning and postulational thinking attest to its pragmatic approach.

Although this book contains the bulk of material usually found in similar texts, the order of presentation is somewhat different. The topic of similarity has been placed close to those of congruence and parallelism. The circle, loci, and regular polygons are treated in the final chapters. Co-ordinate geometry is integrated with the topics of polygons, similarity and the Pythagorean Theorem. The indirect method of proof is developed logically and simply.

Each chapter contains an outlined summary in bold type and a comprehensive two-part review.—R. W. MOLLENDORF, McKinley High School, Chicago, Illinois.

Scholarships, Fellowships, and Loans. By S. Norman Feingold. 254 pp. Bellman Publishing Company, Inc., 1949. $6.00.

This book is highly recommended for reference purposes to libraries and counseling agencies. Many parts of it will be of direct use to the high school and college student. To disseminate information which, when brought to the attention of boys and girls of real ability but of limited means, will make it more possible for them to obtain college and vocational training is the author's purpose. Mr. Feingold is Executive Director of the Jewish Vocational Service of Greater Boston, a community counseling and placement service.

The major part of the book is devoted to rather complete but brief sketches of scholarships, fellowships and loans. These are listed alphabetically by administering agencies. Nearly 300 agencies from the American Academy in Rome offering School of Classical Studies Rome Prize Fellowships to Zonta International offering the Amelia Earhart Scholarship for Women are included.

To make the information more readily accessible there is an Index A of the author's foreword and the section on Planning Your Career; an Index B of names of administering agencies and of the scholarships, fellowships, and loans; an Index C of the scholarships, fellowships and loans listed under vocational goals or fields of interest. In Index C, five scholarships are included under Mathematics with many more in related fields, and thirty-five under Teaching.—JOHN MAYOR, The University of Wisconsin.

THE PRESIDENT'S PAGE

AMONG the many activities of the National Council there is one I wish to call to your attention at this time. You will be interested because it concerns every teacher of mathematics. All of us, as teachers, are working for the improvement of our teaching and, being professional, we give of our ideas and our efforts to help the mathematics situation in general. Realizing the futility of working alone on our problems, we seek the cooperation of others and join one or more mathematics organizations. We do this with the hope of getting help on our teaching problems and to gain some inspiration for the job we are trying to do.

Soon we are asking why the organization to which we belong does not carry on a constructive program and why they do not do something about the matters in which we are interested. We want more from the organization than merely the exchange of ideas, good and helpful as these may be. We want more than just the opportunity of attending a meeting once or twice a year. We are asking for a working organization, one that is composed of people like ourselves who are willing not only to give ideas but are also willing to put forth some effort to see them realized. Furthermore, we want cooperation of organizations, local with national. We know that the improvements we want cannot be realized by the individual working alone nor can much be accomplished by local groups working alone. We want an affiliation of our local group with a state and national organization whose leadership and services will make possible effective work of all groups over the country. We are asking for unity of effort from the bottom on up, as it were.

This is what I wish to call to your attention. We do have exactly what we are asking for. The National Council does have in operation a good plan of group affiliation. It has good leadership and has great possibilities. There are more than forty groups from all over the country that are affiliated with the National Council; most of these are well-organized active *state* groups. These affiliated groups sent delegates to their First Delegate Assembly in Chicago last April. Plans were laid for greater activity on the part of groups and the National Council. At the summer meeting last August, the presidents of seven near-by state groups had an open discussion on ways and means of cooperation and coordination of activities among the groups themselves. Many possibilities were disclosed. At the Christmas meeting in Florida this month, a similar forum will be held, and another will follow at Pittsburgh next March. In fact, it is the plan to have such forums of affiliated group's at all of our National Council meetings.

Eventually, affiliated groups will be more effective in the work they are trying to do and the National Council will be in a position to render the maximum of services to groups and, hence, to the individual members. All of which means that more will be accomplished with much less waste of time and effort. The help of the individual teacher will strengthen the local organization and, in turn, this will make the National Council more effective and more forceful in its endeavors.

To be most successful in our efforts to improve affiliation, we need the help of every member of the National Council. I want to ask something of you. Identify yourself with your state group, if there is one, and become active in it. If there is no group in your state, help to organize one. Offer your help to the State Representative of the National Council in your state. A list of these was published in the October issue of THE MATHEMATICS TEACHER. If your group is not affiliated with the National Council, try to get them to do so. Write the chairman of affiliated groups, Mr. J. R. Mayor, The University of Wis-

consin, Madison 6, Wisconsin, for information on group affiliation. I am asking you, as a member of the National Council, to help in every way you can to make group affiliation really successful. If your state group *is* affiliated, do what you can to help in some definite way. At least, help boost the membership in your group.

It is my prediction and my hope that the work of the National Council through its affiliated groups will prove to be one of its most important activities, an activity that will benefit its members most substantially, and a definite means of giving services to teachers of mathematics all over the country.

H. W. CHARLESWORTH, *President*

AFFILIATED GROUPS TAKE PART IN NATIONAL COUNCIL MEETINGS

JOHN R. MAYOR
Chairman, Committee on Affiliated Groups,
University of Wisconsin, Madison, Wisconsin

FIVE Affiliated Groups of The National Council of Teachers of Mathematics had an active part in the Tenth Summer Meeting of the Council held at The University of Wisconsin, August 21–24, 1950. The Illinois Council of Teachers of Mathematics, the Iowa Association of Mathematics Teachers, the Indiana Council of Teachers of Mathematics, and the Minnesota Council of Teachers of Mathematics planned in advance and participated in specific sections of the program. Presidents of these groups served on the Program Committee for the meeting and were among those presiding at sessions. The Wisconsin Mathematics Council served as host organization for the meeting.

The Iowa and Minnesota groups reported on curriculum studies in their states. "Adjusting the Junior High School Curriculum in Light of Recent Trends in the Elementary Schools" was the topic of the Iowa Group while the Minnesota teachers spoke on "Plans for a New Curriculum in Minnesota."

The Indiana Council sponsored a panel discussion on "The Changes in Geometry" in keeping with the emphasis on geometry to be found in the whole program. Teaching aids that can be used to make mathematics more meaningful and interesting to every student formed the central theme of the Illinois Council's contribution which was presented in three sections—elementary, junior high school, and the general mathematics area.

A discussion of how Affiliated Groups can cooperate for the improvement of mathematics teaching was led by Harry W. Charlesworth with presidents of the state councils of Illinois, Indiana, Iowa, Kansas, Minnesota, Nebraska, and Wisconsin serving as Discussants. As one outgrowth of this session an exchange list for publications of Affiliated Groups was started. If your group was not represented at Madison and would like to be included on the exchange list, please notify the chairman.

The president of the Mathematics Division of the Alabama Education Association, the newest group to become affiliated with the Council, was introduced and spoke briefly of activities in her state.

In order to carry out the recommendation of the First Delegate Assembly that a

part of each meeting of the National Council be devoted to a discussion of the work of Affiliated Groups, panel discussions similar to the one at Madison are planned for the University of Florida Christmas meeting and the annual meeting in March in Pittsburgh. Presidents of Affiliated Groups in the southern area have been invited to take part on the Florida panel. These groups are: Mathematics Division, Alabama Education Association; Florida Council of Teachers of Mathematics; Dade County, Florida, Association of Senior High School Teachers; Hillsboro County (Florida) Mathematics Council; Mathematics Division, Georgia Educational Association; Louisiana-Mississippi Branch of the National Council of Teachers of Mathematics; Mathematics Section, East Tennessee Education Association. Affiliated Groups in Maryland, New Jersey, New York, Ohio, Pennsylvania, Virginia, and West Virginia have been invited to name a representative for the Pittsburgh panel. A more complete announcement of this program will be made in a later issue.

Active and cooperative participation in the programs of the National Council, especially by groups in the areas in which a meeting is held, is one way in which the Affiliated Groups can have an important place in the life of the National Council and also a way in which those in neighboring areas can work and plan together. Advantages of affiliation should come from association with neighboring Affiliated Groups as well as with the National Council. The Committee on Affiliated Groups is anxious to foster more of this cooperative planning and sharing of activities and ideas among Groups. There should be many opportunities for discussion and exchange of teaching materials, and curriculum and research studies.

Plans for a New York State Mathematics Teachers Association

The Committee on Affiliated Groups is happy to receive and glad to publish the following announcement from a New York Committee interested in state organization. Similar requests from other groups, for publication in future issues of The Mathematics Teacher, will be gladly received. Miss Elaine Rapp of New York writes:

> "For several years there have been expressed desires that the mathematics teachers get together and form a New York State association. We have many problems to discuss,—some unique to our own state, some from a national outlook, and some just about teaching.
>
> "At the Chicago meeting of the National Council of Teachers of Mathematics, a group of New York Staters decided to start the ball rolling.
>
> "Approximately 800 letters have been sent to various high schools throughout New York State. We are attempting to compile a list of mathematics teachers who are interested in a State Mathematics group. Obviously, it is impossible to contact every teacher of mathematics; so we are appealing to you now.
>
> "If you are interested in this endeavor, won't you drop a card or letter to one of the committee members listed. We need your support!
>
> Alice M. Reeve, South Side High School, Rockville Centre, N. Y.
> Frances Burns, Oneida High School, Oneida, N. Y.
> Helen Kelly, Fayetteville, New York
> Elaine Rapp, Senior High School, Oceanside, N. Y."

Preliminary Announcement of Speakers Bureau

The Committee on Affiliated Groups is making plans to establish a Speakers Bureau, as recommended by the First Delegate Assembly. The assistance of readers of The Mathematics Teacher will be appreciated. Please send your suggestions to Miss Mary C. Rogers, 462 North Avenue East, Westfield, New Jersey.

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(*Continued on page 435*)

KNOW YOUR NATIONAL COUNCIL REPRESENTATIVES

By Kenneth E. Brown

Chairman of State Representatives, Department of Mathematics, University of Tennessee, Knoxville, Tennessee

Georgia Representative

Miss Bess Patton, an outstanding secondary school teacher, has spent many years in the high schools of Atlanta where she is liked and respected by both teachers and students. For twenty-five years she has been a member and an active supporter of the National Council of Teachers of Mathematics. Her broad training in education at the Teachers College, Charleston, Illinois, and foundation in mathematics at the University of Chicago, has given her a deep insight into the problems of mathematics education. Plans are being made for a large number of the Georgia teachers to attend the Christmas meeting of the National Council of Teachers of Mathematics at Gainesville, Florida. Miss Patton's present address is 443 Ponce de Leon Avenue, Atlanta, Georgia.

Miss Bess Patton

Mr. J. Eli Allen

Alabama Representative

For more than thirty years Mr. J. Eli Allen, Head of Department of Mathematics at Phillips High School, Birmingham, Alabama, has been an inspiration to high school students studying mathematics. His interest in the improvement of the teaching of mathematics has been shown by his active participation in the National Council of Teachers of Mathematics for a quarter of a century. He has contributed articles to such professional journal as *The Peabody Journal of Education*, *The Southern Association Quarterly*, and The Mathematics Teacher. Last year he sent more than one hundred letters to teachers of mathematics asking them to join the National Council of Teachers of Mathematics. In addition to such invitations, this year he plans to suggest that they spend part of

their Christmas vacation at the meeting of the National Council at Gainesville, Florida. For all his efforts, his only remuneration has been the satisfaction one gets from serving a cause to which one is devoted. This is the policy followed by all Council Representatives. Mr. Allen would welcome the assistance of any teacher in Alabama in helping him with his statewide National Council publicity program.

South Carolina Representative

Miss Lucile Huggin, Spartanburg, South Carolina, has taught in the public high schools of South Carolina since 1935. During the last three years she has been the National Council Representative. Her academic education was received from several colleges. She received the A.B. degree from Winthrop College, the M.A. from Columbia University, and she has done additional graduate work at the University of South Carolina, University of North Carolina and Peabody College for Teachers. It is expected that a large number of the teachers of mathematics from South Carolina will join with her in attending the Christmas Meeting of the National Council of Teachers of Mathematics at Gainesville, Florida.

Miss Lucile Huggin

(*Continued from page 433*)

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ATTENDANCE RECORD OF THE TWENTY-EIGHTH ANNUAL MEETING CHICAGO, ILLINOIS—CONGRESS HOTEL—APRIL 12-15, 1950

Compiled by EDWIN W. SCHREIBER, *Secretary*
State College, Macomb, Illinois

* Member of the National Council of Teachers of Mathematics.
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Sullivan
Hart, Helen J.
McIlvain, Leta
Taft
*Ryan, Mary F.
Tilden Technical
*Bradfield, George F.
*Garas, Myrl
Minskl, Daniel (St)
*Simcox, Mabel
*Woessner, Anne L.
*Puder, Maud J.
Tulley
*Clyne, Margaret M.
Von Steuben
Dolins, Gloria J. (St)
Heytow, Eugene
*Hewitt, Glenn F.
*Idtse, Theodora
Wells
Frederick, Leo
*Leifel, John F.
*Marek, Emily
*Urbancek, Francesca

College

Chicago Teachers College
Aurand, Joyce H. (St)
Bailey, Cleo (St)
Barber, Barbara T. (St)
Bertoia, Gloria S. (St)
Bornath, Katharine B. (St)
Borowski, Eleanore (St)
Byrnes, Margaret (St)
Cachey, Mary T. (St)
Cook, Raymond M. (V)
*Coyne, William J. (St)
Dahlberg, Dorothy (St)
DeBofsky, Jean (St)
DeGrange, Lois J. (St)
DeSimone, Phyllis (St)
Grandsart, Vivianne (St)
Groetsema, Helen (St)
Jenson, Jantena E. (St)
Jorgensen, Erlinga (St)
Kloman, Elizabeth (St)
Koehl, Regina T. (St)
Kostris, Marceline (St)
Kubilius, Rita (St)
Luchich, Dragana (St)
Marciante, Marie (St)
McKinney, Virginia (St)
McNally, Mary (St)
Meenahan, Mary (St)
Meyer, Betty (St)
Mitchell, Patricia (St)
Moldenhauer, Rita (St)
Outly, Eleanor J. (St)
Peckerman, Lillian (St)
Rakow, Alyce M. (St)
Sachs, Jerome M. (St)
Sancher, Manuel (St)
Szulakiewicz, Delphine C. (St)
Truesdale, Dolores (St)
*Urbancek, Joseph J.
Walsh, Marjorie A. (St)
Waro, Francis G. (St)
Woelkers, Jane F. (St)
Woods, Margaret (St)
De Paul University
*Svoboda, Arthur
Herzl Junior College
*Kurzin, Wm. H.
*Piety, Harold M.
Illinois Institute of Technology
*Bibb, Sam F.
*Comfort, Edwin G. H.
Ford, Lester R.
Hagensee, Theodore E.
Mundelein
Murphy, Helen (St)
North Park
*Backlund, Linnea M.
*Lindberg, Viola G.
*Ludwig, Ruth
Roosevelt
Goldman, Gordon
Gore, G. D.
Street, Alan T.
St. Xavier
Basile, Antoinette T.
*Sister Mary Elaine, R.S.M.
Sister Mary Charlotte, R.S.M.
*Sister Mary Dulciosa
University of Chicago
*Breslich, E. R.
Forsythe, Albert
*Fouch, Robt. S.
*Hartung, Maurice L.
Jensen, Richard
*John, Lenore S.
*Layne, Ray M.
Levine, Marilyn F.
Nichiporuk, Eugene
University of Illinois
Berglund, Winifred V.
Corliss, John
Davis, James E.
*Feinstein, Irwin K.
*Hartley, M. C.
*Hornacek, Rose L.
Iverson, Alice F.
*Nowlan, Frederick S.
*Ondrak, Thomas B.
Wilson, Chas. Craig
Wilson Junior College
Burack, Marvin (St)
*Feltges, Edna M.
Gutekunst, Hans
Horwitz, David (St)
*James, Howard A.
Kennedy, Elmer
Potin, Henry
*Stein, Norman
Williams, O. S.
Wright Junior College
*Buelow, Elsa Muuss
*Bulinski, Richard L.

Catholic Parochial Schools

Good Counsel
Sister Mary Sanctoslaus
Sister M. Elna, Fel
Sister M. Gerald
Loretta
*Mother M. Alurada
Holy Family
Sister Mary Lauretine
*Sister M. Gemma
Holy Trinity
*Brother Reginold
Imaculata
*Sister Mary Eileen
*Sister Mary Timothea
St. Ann
Sister Ludvina
*Sister Mary Canisia
St. Casimir
*Sister M. Pulcheria, S.S.C.
*Sister M. Virgilia
St. Joseph
Sister Mary Lucetta
Sister Mary Benilda
St. Mary
Sister M. Zenobia, S.S.J.
Visitation
*Sister Florentinus

Lutheran Parochial Schools

Holy Cross
Reuter, John P.
St. Paul's
Kuester, Roberta M.
Kurth, Edward
Peters, William A.
Tetting, William H.

Others

Ballenger, William
*Berg, Hazel S.
Bishop, A.
*Brown, Dalna
Cohen, A. C.
Fitzgerald Matthew
Forrest, Daisie Ella
Harrell, Blondelle
Hoffmann, Valerie
Howell, Lillian W.
Greganti, Jane
*Hainbecker, Lucy E.
Jones, Jimmie Lee
Ladendorf, Dorothy J.
Leckrone, Thomas
Rappaport, Mrs. David (V)
Redman, David A.
Rosenberg, Edward
*Sister Oliva
Strauss, Raymond C.
Walsh, Wm. J.
Woerner, Ruth
Young, Roy

Exhibitors

Butler, A. F.
Clark, Herbert

Cottle, William R.
Davis, Nelson
Engle, Kenneth
*Ferguson, J. C.
Flemming, George E.
*Greene, Wesley
Griffin, Lee H.
Kahler, Laura H.
Kennedy, Rod E.
Kraus, W. L.
Martin, Lloyd J.
McNamara, Jr., Robert
Mathers, Clay B.
North, Aubert
*Russell, George E.
Stanton, Walter S.
Scholz, Marie
Schultz, E. H.
Shiras, Sylvia
Snyder, E. Grant
Trione, Phyllis
Uhrig, Jack E.
*Young, Arthur T.
Chicago Heights
*Briggs, Ralph F. (H)
*Morrison, Gladys (H)
Cicero
*Cherry, William J. (H)
*Dobbes, Marie A. (E)
*Elam, Cecil W. (H)
Kouba, Norman G. (H)
Ledbetter, Mary (H)
*Miller, Mabel I. (H)
*Reeder, Ruth C. (E)
*Richards, William A. (J C)
*Royce, Geo. L. (H)
*Sistler, Hobert (H)
Tintera, George B. (St)
*Tucker, Alice N. (J C)
*White, Harold J. (J C)
Woodrich, Kenneth (St)
Clarendon Hills
Shirk, Raymond E. (H)
Clinton
*Marshall, Opal (H)
*Whitmore, E. H. (C)
Crystal Lake
*Ackerman, Vernon (H)
*Tingleff, Howard C. (H)
Danville
Hitchens, Emma J. (E)
Hitchens, Hazel (E)
Ingram, Mildred B. (H)
Spries, Edith (E)
Decatur
*Blackford, Clarence (J)
*Fisher, Georgia H. (H)
*Gray, Kathryn L. (J)
*Herbert, M. Cecilia (H)
Huddleston, Roy L.
Layton, Lovick (U)
*Maurer, J. J. (J)
*Prestley, Margery (H)
*Taylor, Clarence E. (H)
Westerman, Ralph B. (H)
*Williams, Frances A. (H)
De Kalb
*Helmich, Eugene W. (C)
*Helming, Carolyn (H)
Morton, Frank G. (H)
Paine, Edward L. (H)
*Stelford, Norma (C)
*Storm, Wm. B. (C)

Des Plaines
Pepin, Frances (J)
West, Ella B. E.
Dixon
*West, Ada H. (H)
Downers Grove
Bennett, Joseph C. (H)
*Clark, Mildred I. (H)
Cressey, Ralph E. (H)
Manning, Grace L. (J)
*Miller, J. Lee (H)
Dupo
*Glascock, Lucy (H)
Sachs, Leroy (H)
Dwight
Carter, Mary Lou (H)
Wepprecht, Verna C. (J)
East Peoria
DuBois, Robert W. (H)
Foote, Frances
*King, George Y. (H)
East St. Louis
Alex, Cecilia H. (H)
Gagnon, Alphe
*Hubbard, Elbert C.
Edwardsville
*Alexander, Nellie L. (H)
Elgin
Ackermann, Stella (E)
Owen, Elizabeth (J)
*Peters, Mary (H)
*Siren, Theresa A. (H)
Sullens, Beulah (E)
*Thom, Adela M. (C)
*Wilson, Hortense E. (H)
Elmhurst
Baumgart, John Y. (C)
*Settle, Ida Lane
Elmwood Park
*Janicki, George J. (H)
Evanston
*Anderson, Neva E. (H)
*Anspaugh, Robert E. (H)
Bradfield, Mrs. G. F. (V)
Barringer, Barbara (St)
Bergman, Margaret (St)
*Bradley, Jean A. (H)
Burgess, Willis W. (St)
*Cady, Doyle (H)
Cascino, Lee R. (St)
Dummer, James H. (St)
*Exley, Helen M. (H)
Haswell, Janet S. (St)
Hathaway, Arthur S. (C)
*Hildebrandt, E. H. C. (C)
Hildebrandt, Mrs. E. H. C. (V)
*Iverson, Donald W. (H)
*Johnson, John T. (C)
Knopf, William C. (St)
*Koepnick, Frederick A. (H)
*Leach, Edgar S. (H)
Leach, Mrs. Edgar S. (V)
Merrick, Marion M. (St)
*Moorhouse, Drucelia (H)
Moss, Wilfred A. (St)
*Norberg, Arthur (J)
Petefish, Howard M. (St)
Potts, Floyd (St)
Rasmus, Mildred H. (E)
Reardon, Blanche (St)
Rinehart, Clifford F. (St)

Sandstrom, Herbert (St)
*Sauer, Herbert L. (H)
*Stafford, Acenith V. (H)
Tecotzky, Bernard (St)
Umberger, Alfred A. (St)
*Wildermuth, Karl P. (J)
Wilson, James S. (H)
Forrest
*McWherter, Edwin M.
Forest Park
Scheiwe, Arthur W. (J)
Franklin Park
*Weston, Virginia C. (H)
*Flood, Elizabeth F. (H)
Tripp, Muriel R. (H)
Freeport
*Baumgartner, R. A. (H)
*King, Alice (J)
*Kuhlemeyer, Ferne (H)
*Martin, Mary C. (H)
*Rubendall, W. C. (H)
*Sullivan, Irene V. (H)
Galesburg
*Roberts, June L. (J)
Glencoe
*Montgomery, A. L. (E)
Glen Ellyn
*Carlson, Ruby A. (J)
Ernst, Margaret M. (J)
Godfrey
*Hall, Cleo (C)
Granite City
*Fisher, Gussie L. (H)
*Huck, Lucinda L. (J)
Granville
*Whitaker, Wilhelmina S. (H)
Gurnee
*Benjamin, Edith W. (H)
Harvey
*Hanschmann, Alice (H)
*Jensen, Ruth A. (H)
Harvard
*Leslie, Donald P. (H)
Highland Park
*MacMartin, Christine H. (H)
*Weldin, Marie (H)
Hinsdale
*Miller, Barbara J. (H)
*Takala, Reino M. (H)
Hoopeston
Taylor, Miriam (H)
Jacksonville
*Hollowell, John M. (H)
*Shumaker, John A. (C)
Joliet
*Aseltine, Leland B. (H)
Enderson, Harris (H)
*Fisher, Mary Louise (H)
*Longman, Beryle K. (H)
Kankakee
*Endsley, Elizabeth S. (C)
*Hopkins, Mildred (H)
*Ingli, Gwendolen (H)
Keithsburg
*Kiddoo, Maude (H)
Kewanee
Astrauski, Catherine M.
Kimmell, Elsie M. (H)
*McCarthy, Harriet (H)
Watson, Archie M. (J)
Kinderhook
*Miller, Elsie M. (H)

La Grange
*Allen, Frank B. (H)
Gundry, Helen (E)
*Hawkins, George E. (H)
McKinnon, Nettie J. (E)
*Montgomery, Forest (H)
*Nielsen, Mildred R. (J)
Prater, Belle (E)
*Schneider, Helen A. (JC)
*Sister M. Constance (H)
*Sister Joan Marie (H)
LaSalle
*Ennor, Tirza (C)
Lawrenceville
*Roth, Selma (H)
Libertybille
*Hart, W. W.
Lincoln
*Henneberry, Theresa (H)
Lockport
*Aiken, Daymond J. (H)
*Goerz, Lormia E. (H)
Macomb
*Alexander, Robt. T. (H)
*Ayre, H. Glenn (C)
Cox, Irvin (St)
Lundahl, Arthur (St)
Sage, James (St)
*Schreiber, Edwin W. (C)
Schreiber, Mrs. E. W. (V)
Sloboda, Joseph (St)
Smith, Shelby (St)
*Stipanowich, Joseph (C)
Subject, Paul (St)
*Taylor, Loren F. (C)
Mattoon
*Albers, Glenna J. (E)
Maywood
*Arends, Lillian (H)
*Baer, F. W. (H)
*Foster, U. S. (H)
*Hildebrandt, Martha (H)
*Joyner, Earle L. (H)
*Kent, Vernon R. (H)
*Law, Wilson A. (H)
*Manilaw, Harold G. (H)
Morris, Emmet (E)
Peters, Hugh M. (H)
*Scheible, Mabel S. (H)
*Sims, Wima T. (H)
*Sullivan, Margaret (H)
*Terhune, Virginia (H)
Melrose Park
Golden, James E. (J)
Kennedy, Minnie W. (E)
McGillicuddy, Ellen (E)
Melvin
Fawkes, Clayton A. (H)
Moline
*Merwin, Jack C. (J)
Monmouth
*McKeown, Marjorie (H)
Monticello
*Fleming, Mildred D. (H)
Morris
*Johnston, Nellie M. (H)
Mt. Carmel
*McLaughlin, Clara (H)
Mt. Vernon
*Richardson, Eleanor (H)
*Wood, Velma S. (H)
Mt. Pulaski
*Vandevender, W. H. (H)

Naperville
Moore, Thomas P. (C)
Seybuld, Anice (C)
Newman
*Rambis, Albert (H)
Normal
*Flagg, Elinor B. (C)
Lichty, Robert E. (C)
*Norskog, Edna M. (C)
Smith, Ralph E. (C)
*Ullsvik, Bjarne R. (C)
Northbrook
*Gallagher, Clarence (H)
*Marwick, Julia R. (J)
Richards, May E. (H)
Oak Lawn
*Cervenka, Elaine R. (E)
Oak Park
Foster, Frank M. (H)
*Hartman, Mary Jane (H)
*Johnson, Elsie (H)
Rapp, Katherine H. (H)
Rapp, John H. (V)
Van Dyke, John K. (H)
*Woodruff, Robert S. (H)
Ottawa
*Hodgson, Elsie G.
*LeMay, Mary R. (H)
Palatine
Cutler, Edith L.
*Gibbs, Mae M. (H)
Palos
Foyer, Elizabeth G. (E)
Tierney, Alice M. (E)
Palos Park
Braasch, Olive P. (E)
Brennan, Leona M. (E)
Henry, Lula H. (E)
Johnson, Gabrielle (E)
Kirsch, Tracy Ann (E)
Myland, Mary C. (E)
Schlieben, Ray L. (E)
Smead, James (E)
Sparks, Joy C. (E)
Paris
Rhodes, Doris P. (H)
Park Ridge
*Staiger, Herman T. (E)
Pekin
*Blair, F. Mae (H)
*McCoy, M. Eleanor (H)
Peoria
*Albright, Ada Mae (H)
*Bielema, Martin M. (H)
Carline, W. G. (H)
*Deal, John W. (H)
Giles, Florence I. (H)
*Martens, Mildred M. (H)
Polo
DePrino, Louis D. (J)
Quincy
Velesz, Rev. Dunstan (C)
River Grove
Moore, Alice R. (J)
River Forest
Baumgart, E. (St)
Behrens, Ernest (St)
Berhorst, E. (St)
Bergman, Fred (St)
Buckley, Therese A. (St)
Christiansen, Glenn (St)
Conahan, Agnes M. (St)
*Davis, Elsie L. (J)
Doehrman, V. (St)

Frillman, J. (St)
Gabriel, Louis (St)
Gugel, Edna (St)
Hendrickson, Don (St)
Hoehne, Emil (St)
Holdorf, Ruth (St)
Junghans, E. (St)
Koschman, Helen (St)
Kuhlmann, Antoinette (St)
Laabs, Charles (St)
Luedtke, F. (St)
Luke, Frances (St)
Meier, Margaret (St)
Meier, Marilyn (St)
Pelange, Richard A. (St)
Pester, Kathleen A. (C)
Pieper, Martin C. (C)
Pieper, Theodore M. (St)
Pinnow, A. (St)
Pollard, Dolores
Rechenmacher, Mildred (C)
Reitmeyer, Royce (St)
Rund, Josephine M. (C)
Runge, Ethel (St)
Rusch, Grace A. (C)
Schabel, R. (St)
Schaeffer, O. (St)
Seeboldt, E. (St)
*Sister M. Phillip Steele (C)
*Sister Marie Stephen (C)
Suhr, E. (St)
Tierney, Agnes M. (St)
Voll, Eugene (St)
Warnke, F. (St)
Winkelman, Audrey (St)
Zorn, Marie (St)
Riverside
Arthur, Lee E. (H)
Courtney, Nettie K. (H)
Duval, Warren L. (H)
Stanger, George H. (H)
Rockford
Brown, Betty J. (J)
Burchfield, Mary (J)
*Erb, Russell J. (H)
*Hollem, Ruth M. (J)
*Johnson, Chester L. (J)
*Martin, Ruth B. (J)
Penstone, Florence P. (H)
*Presnell, Roberta E. (C)
Ragsdale, Ralph L. (J)
Sheetz, E. Christine (H)
Slade, Katherine (H)
Rock Island
Norton, Donna L. (E)
Round Lake
Cox, Mary Edna (H)
Serena
Leer, Martha L.
Shabbona
*Boon, Alice (H)
Skokie
deBooy, Margaret (H)
Gallagher, Valerie Z. (H)
Springfield
Barrick, Beulah B. (H)
*Campbell, Lorene E. (H)
*Chatburn, Frances (H)
*Clapper, Sadie E. (H)
*Clark, Roy (Su)
*Mason, B. Irene (H)

*Parker, Merle R. (H)
Streator
Cobb, Edward L. (H)
*Ferguson, Florence (H)
*List, Earle B. (H)
Lloyd, Lewis D. (H)
Urbana
Bauer, Marie L. (H)
Birr, Donald (St)
*Gill, Billie (H)
*Henderson, K. B. (C)
Hudgins, Donald R. (C)
*Meserve, Bruce E. (C)
Mock, Gordon (St)
*Nelson, Agnes L. (H)
*Neuss, Vivian Rose (C)
*Pingry, Robert E. (C)
Roelig, Kenneth (C)
*Snader, Daniel W. (C)
*Willoughby, D. S. (C)
Wormood, Edith (St)
Wauconda
Zar, Julian L. (H)
Waukegan
Anderberg, Glenn (H)
*Barczewski, Walter (H)
*Clymer, Francis P. (H)
*Dady, Bess L. (H)
*Grady, Florence (H)
*Greenleaf, Myrtle (H)
Larson, Victoria L. (H)
Melton, Charles E. (H)
*Wright, John R. (H)
West Chicago
*Balzhiser, Marie W. (H)
*Horn, Clara E. (J)
Wheaton
Brandt, Angeline J. (C)
Wilmette
*Gelbert, Elizabeth (J)
*Wyman, Herma (J)
Wilmington
*Crone, Roy L. (H)
Winnetka
Dawson, Laura (E)
*Evans, Ethel M. (H)
*Funkhouser, David (H)
*Jones, C. Herbert (H)
Scoyes, Lela (J)
Smith, Don F. (V)
*Swain, Henry (H)
*Taylor, Lewis A. (H)
*Ude, Norman E. (H)
Wood River
*Love, John J. (H)
Zion
Klinge, Muriel H. (H)
*Krughoff, Florence (H)
*Studer, Mae E. (H)

Indiana
Anderson
*Ahrendt, M. H. (C)
Bloomington
*Peak, Philip (C)
*Shetler, Luther L. (C)
Crawfordsville
*Biddle, Pauline W. (H)
*Smith, Mabel T. (H)
Crown Point
Cochran, Alton W. (H)
Spangler, Mamie C. (E)
Culver
Donnelly, Alfred J. (H)
Gowan, John C. (H)
Stinchcomb, Judd T. (H)
East Chicago
*Brennecke, Marie (H)
*Duncan, Naomi (H)
*Kauffman, Geraldine
*Johnson, Florence M. (H)
*Neal, Sadie M. (H)
Wall, Ethel (E)
*White, Eva A. (J)
Elkhart
Boone, Zella L. (J)
Likins, Daisylind S. (J)
Minterl, Luella (J)
Evansville
*Meyer, Henry A. (H)
Fort Wayne
Danuser, Virginia D. (J)
*Fiebig, Elmer F. (H)
*Fiedler, Adelaide L. (H)
Hall, Velma J. (E)
Miller, Marie (H)
Plumanns, Leona (J)
*Wear, Olive G.
Frankton
*Conkling, Kenneth (H)
Conkling, Annabelle (V)
Gary
*Flewelling, Wilma S. (H)
Green, Bernice C. (J)
*Gwinn, Adele (H)
Hilbig, Herman G. (E)
*Leskow, Olive (J)
*Rzepka, Helen (H)
*Stewart, Leonora W. (H)
*Swank, O. M. (H)
*Thompson, Roger (H)
*Waggoner, Olive E. (E)
Wallace, Frank (Ex)
Goshen
*Graham, Eva J. (H)
Hartzler, H. Harold (C)
*Schenck, Stanley (H)
Hammond
*Abell, Thelma L. (J)
*Bollin, Alice M. (J)
*Brock, Robert F. (H)
Cleveland, Alice M. (H)
Garrett, Chas. G. (Su)
Glegoroff, William A. (E)
Hanlon, Margaret E. (H)
*Lundgren, Lawrence (H)
*Snadden, Leonard J. (J)
*Steiner, Gertrude V. (J)
*Williams, Katherine (H)
Hartford City
*Allee, Kenneth O. (H)
Indianapolis
*Belding, Robert (H)
*Elliott, Mabel (J)
*Gee, William S. (H)
*Pearson, Helen (H)
*Welchons, A. M. (H)
*White, Anna L. (H)
*Wilcox, Marie S. (H)
Lafayette
*Beck, William R. (C)
*Carnahan, Walter H. (C)
*Pritchard, Ralph (C)
*Wirsching, Florence (C)
La Porte
*Hogle, Charlotte (H)
*Knight, Azalia A. (H)
Lebanon
*Beckett, K. Eileen (H)
Michigan City
Giffel, William (J)
Kwiatkowski, Ralph (H)
*Lockridge, Elbert (J)
Muncie
*Carr, Alice Rose (C)
*Edwards, P. D. (C)
*Whitcraft, L. H. (C)
No. Manchester
Dotterer, John E. (C)
Pierceton
*Mowrey, John F. (H)
Plymouth
*Trowbridge, Julia (H)
South Bend
*Kitson, Mary A. (H)
Sister M. Francis (H)
Sister M. Rosalima (H)
Terre Haute
*Blaikie, Muyrel M. (J)
*Kelly, Inez (H)
*Kennedy, Kathryn (H)
*Lundford, Rosella F. (J)
*Moore, Vesper D. (C)
*Strong, Orvel E. (C)
Valparaiso
*Pauley, Claude (C)
Whiting
*Booth, Leah (H)
*Canine, Margaret E. (H)

Iowa
Ames
*Miller, Ruth (H)
Burlington
*Houston, Lucille P. (J)
Britt
*Muller, Elsie (C)
Cedar Falls
*Abbas, Lena I. (C)
*Allender, Robert E. (C)
*Brune, Irvin H. (C)
Brune, Mrs. Irvin H. (V)
Edwards, Donald R. (C)
*Gibb, E. Glenadine (C)
*Kearney, Dora E. (C)
*Keppers, George L. (C)
*McClure, David S. (C)
McGrew, James (St)
*Richardson, Donald (C)
*Sage, Eddie E. (C)
*Van Engen, H. (C)
*Whiting, Donna L. (C)
Wiegert, Samuel C. (C)
Charles City
*Hinsenbrock, Kathryn (H)
Clinton
Herkelmann, Leo E. (H)
Des Moines
*Davison, Ruth H. (H)
McEniry, Margaret (H)
Tallmann, Bertha M. (J)
Dubuque
Rothisberger, Hazel (C)
Sister M. Gervase (E)
*Sister Pauline (H)
Steichen, Kathryne (J)
Independence
*Wollesen, Marlys (H)
Iowa City
*Price, H. Vernon (C)

*Spitzer, Herbert F. (C)
Marshalltown
*Christensen, Amanda (J)
*Cornwall, Marion (J)
Mason City
*Chapdelaine, Perry (J)
Mt. Pleasant
*Walker, Mabel (H)
Storm Lake
Roorda, Ethel (C)
Waukon
*Frederick, Dorothy (H)
Waverly
*Wiederanders, Richard (J)

KANSAS
Atchison
Culivan, Jeanne M. (St)
Donlon, Frances A. (St)
Emporia
*Otterstrom, Ruth E. (C)
Lawrence
*Ulmer, Gilbert (C)
Topeka
*Henderson, K. June (J)
Lake, Kenneth (St)
Wichita
*Hall, Lucy E. (H)
*Nickel, Kenneth N. (H)
*Richert, Anton S. (H)

KENTUCKY
Anchorage
*Wood, Edith L. (H)
Louisville
*Cowan, Florence J. (J)
*Eccleston, Stella (J)
Ford, W. Clarence (H)
*Hale, Loretta H. (J)
*Metcalf, Teresa B. (J)
*Pettus, Shirley Gill (H)
Pembroke
*Fugua, Elizabeth F. (H)

LOUISIANA
Baton Rouge
*Karnes, Houston T. (C)
Lafayette
*Begnaud, Lurnice P. (H)
*Heard, Ida Mae (C)
McGuire, Norma S.
Natchitoches
*Maddox, A. C. (C)

MARYLAND
Baltimore
*Blackiston, Nannette R. (J)
*Bowers, Eunice (J)
Carroll, Louisa (H)
Clarke, Grace (J)
Fisher, Elizabeth (E)
Hartje, A. Elizabeth (E)
*Heinzerling, Margaret (H)
*Herbert, Agnes (J)
Holzapfel, Gertrude (J)
Hucksoll, Wm. J. (H)
Hucksoll, Mrs. Wm. J. (V)
*Menton, Margaret M. (H)
*Merriken, Ruth (J)
*Parker, Beulah (J)
*Schwartz, Esther (J)
*Taylor, Leroy (H)
Frostburg
*Layton, William I. (C)
Layton, Mrs. William I. (V)
*Rissler, Walter J. (C)
Prince Frederick
*Davis, Bertie W. (H)
Towson
Moser, Harold (C)

MASSACHUSETTS
Boston
*Betts, Barbara B.
*Buckingham, Burdette
*Burch, Robert (C)
Davis, Corinne W. (V)
*DuVall, Dale A. (C)
Fenollosa, George
*Gravel, Martha
*Smith, Geraldine
*Syer, Henry W. (C)
Brookline
*Ward, Ralph (H)
Gloucester
*Parkhurst, W. S. (H)
Hingham
*Ricci, Theodore (H)
Springfield
*Smith, Rolland R. (H)
Smith, Mrs. Rolland R. (V)

MICHIGAN
Albion
*Larsen, Harold D. (C)
Ann Arbor
Anning, Norman (C)
Byers, Orland L. (St)
*Hach, Alice M. (C)
*Jones, Phillip S. (C)
Kyselka, Will (St)
Lohela, Arvo E. (H)
*Noyes, Dorothy E. (H)
Augusta
*Lueker, Helene C. (H)
Battle Creek
Seedorff, Ava M. (H)
*Tamperi, Mary Ann (H)
*Wakefield, Hazel (H)
Bay City
*Anderson, Edith G. (H)
*Ewing, Meta M. (C)
Benton Harbor
*Bolenjack, Hobert (JC)
Brown, Carita A. (E)
Field, Marian (H)
McNutt, Grace (H)
*Reed, Mary E. (H)
Chelsa
*Schell, Esther L. (H)
Constantine
*Harvey, Dorothy B. (H)
Harvey, Josephine (V)
Detroit
*Dolan, Geraldine M. (H)
*Fern, Martin (J)
Fern, Mrs. Martin (V)
*Frey, Franklin (H)
Frey, Mrs. Iness (V)
*Junge, Charlotte W. (C)
Rowlson, Robert
*Sauble, Irene (H)
Thiele, C. L. (J)
East Grand Rapids
*DeJonge, Helen B. (H)
Grand Rapids
*Reese, Dora W. (H)
Grosse Pointe
*Schermer, Bertha M. (H)
Hopkins
Barnes, Barbara (H)
Iron Mountain
*Helming, Dorothy (H)
Kalamazoo
Bartoo, Grover (C)
Bartoo, Mrs. Harriette (V)
*Butler, Charles H. (C)
Cain, Wm. H. (C)
*Everett, John P. (C)
*Ford, Pearl (C)
Gish, Grace I. (C)
*Hannon, Herbert H. (C)
*Meagher, Jack R. (C)
*Meagher, Mrs. J. R. (V)
*Richardson, John A. (C)
*Rogers, William W. (C)
Lansing
Hagen, Verna E. (J)
Johnson, Ruby V. (J)
*Schneider, Russell L. (H)
*Seymour, Mildred I. (J)
Marlette
*Gaskins, Golda Marie (H)
Marquette
*Bjork, Clarence M. (C)
Bjork, Mrs. C. M. (V)
*Boynton, Holmes (C)
Midland
Wang, William A. (H)
Muskegon Heights
Nichols, Josephine (J)
Schregardus, Lucille (J)
Saginaw
Giesecke, Harold W. (H)
Sault Ste Marie
*Joslin, Ethel M. (H)
South Haven
Norlin, Frederick O. (H)
Swanson, Oscar (J)
Sparta
Marshall, Elizabeth (H)
St. Joseph
Roth, Elmer A. (E)
Sturgis
*Brokow, Helen M. (H)
Seitz, Mary G. (H)
Ypsilanti
Schmidt, Beverly (C)

MINNESOTA
Anoka
*Wilcox, Oscar O. (H)
Brainerd
*Kinn, Louise L. (J)
Duluth
*Elwell, Mary I. (C)
Mankato
*Horeni, Hildegarde (C)
Minneapolis
*Brueckner, Leo J. (C)
*Farm, Charlotte (C)
*Johnson, Donovan (C)
*Kellogg, Theodore (H)
*Woolsey, Edith (J)
Northfield
*Wegner, Kenneth W. (C)

Rochester
*King, Ethel (H)
Sinclair, Mary (J)
St. Paul
*Berger, Emil J. (H)
O'Leary, Eve F. (J)
Sister Cornine Carter (H)
*Sister Alice Irene Frieberg (H)
Winona
*Leo, Brother J. (C)
*Lokensgard, Rudolph L. (C)

MISSOURI
Clayton
*Montgomery, Gaylord C. (H)
Kansas City
*Doyle, William C. (C)
Kirksville
*Jamison, G. Harold (C)
St. Louis
Gorman, George W.
Johnson, W. P.
*Marth, Ella (C)

MONTANA
Dillon
*McCollum, Lenore T. (H)

NEBRASKA
Kearney
*Nelson, Theordora S. (C)
Lincoln
Carter, Joey I. (H)
*Clark, Myrtle E. (H)
Omaha
*Barstow, Louise W. (H)

NEW HAMPSHIRE
Exeter
*Adkins, Jackson B. (H)
Keene
*Peters, Ann (C)
Plymouth
*Smith, Geneva M. (C)

NEW JERSEY
Jersey City
*Grossnickle, F. E. (C)
Montclair
*Mallory, Virgil S. (C)
Trenton
*Shuster, C. N. (C)
Westfield
*Rogers, Mary C. (J)

NEW YORK
Albany
*Gardner, Randolph S. (C)
Brooklyn
Klein, Rose (J)
*Schaaf, William L. (C)
Buffalo
*School, Louis F. (Su)
School, Mrs. Louis F. (V)
Canastota
*Daniels, Gertrude E. (H)
Floral Park
Ostrander, Ida A. (H)
Garden City
*Baravalle, Herman (C)
Marcellus
*Brooks, Lucile E. (H)
New York
*Bakst, Aaron (C)
Bakst, Mrs. Aaron (V)
Darcy, George
*Fehr, Howard F. (C)
*Geise, Stanley E.
*Kinsella, John J. (C)
*Neal, Nathan A.
Whitney, Stanton D.
*Wickhavn, Joseph
Oceanside
*Rapp, Elaine (H)
Oneida
*Burns, Frances M. (H)
Oneonta
*Johnson, Frances C. (H)
*Kelley, Helen G. (H)
*Sanford, Vera (C)
Queens
*Rich, Barnett (H)
Rochester
*Betz, William (H)
Rockville
*Reeve, Alice M. (H)
Schenectady
Lee, Everett S.
Syracuse
*Rosskopf, Myron F. (C)
West Point
*Yates, Robert C. (C)

NORTH CAROLINA
Durham
*Rankin, W. W. (C)
*Williams, Annie (J)

OHIO
Ada
*Pfeil, Arthur E. (H)
Alliance
*Freese, Frances (C)
Athens
*Lee, Harold L. (H)
Lee, Mrs. Harold L. (V)
Cincinnati
*Becker, Marie (H)
*Keiffer, Mildred C.
*McNelly, Nanabel (J)
*Struke, Norma L. (J)
Cleveland
*Brumfield, Emalou (C)
*Grime, Herschel E.
Grove, E. L. (V)
*Kraft, Ona (H)
*Miller, A. Brown (H)
Columbus
*Barcos, Howard J. (H)
*Brown, John J. (J)
*Carter, William L. (C)
*Fawcett, Harold P. (C)
*Hardgrove, Mrs. Clarence (C)
*Heinke, Clarence H. (C)
*Lazar, Nathan (C)
*Myers, Sheldon (C)
*Schaaf, Oscar F. (C)
*Schacht, John F. (H)
*Shannow, Pauline (H)
*Smith Eugene P. (H)
*Stalzer, Elsie J. (C)
*Stapleford, Edw. T. (C)
Cortland
*Wall, Florence R. (H)
Dayton
*Miller, Reah H. (E)
Finlay
*Bell, Genelle (C)
Kent
*Harshbarger, Frances (C)
Kenton
*Water, Ralph R. (H)
Mansfield
*Lantz, William B. (H)
Medina
Insprucker, John H. (H)
Middlebranch
*Ream, Doris V. (H)
Oxford
*Christofferson, H. C. (C)
*Schluter, I. L. (C)
Palestine
Wilson, F. M. (V)
*Wilson, Frances P. (H)
Parma
*Grove, Ethel L.
Salem
*McCready, Martha (H)
Shaker Heights
*Bowen, Alma (H)
*Miller, Florence B. (J)
*Selorer, Hattie C. (J)
Toledo
Dancer, Wayne (C)
*Sister M. Constantia (H)
Wyoming
*Naugle, Jacob V. (H)
Naugle, Mrs. Jacob V. (V)
Youngstown
*Rukenbrod, Mary (H)

OKLAHOMA
Norman
*Lewis, Eunice M. (H)
Oklahoma City
*Shike, Virginia C. (J)
Stillwater
*Zant, James H. (C)
Zant, Mrs. James H. (V)
Tulsa
*Lackey, Muriel O. (H)
*Lavengood, L. W. (H)

PENNSYLVANIA
Braddock
*George, Clementina (H)
Bradford
*Downing, Freas (H)
Cresco
*Bosman, Edward E. (H)
Folsom
*Smith, Leonard L. (H)
Greensburg
*Sister M. Deborah Kelly (C)
*Sister Mary Thaddeus (C)
Kutztown
*Knedler, Paul A. (C)
McKeesport
*Richardson, Amelia (H)
Millersville
Child, Kenneth D. (St)
Cocklet, Richard H. (St)
Jones, Alene K. (St)
Keneagy, John H. (St)
Lehman, Florence (St)

Philadelphia
Mellott, Malcolm
*Oliver, Albert I. (C)
Pittsburgh
*Lyons, Catherine (H)
*Sister M. Jane (E)
*Sister M. Tarcicius (H)
Verona
*Baker, Mabel Love (H)

South Dakota
Belle Fourche
Roberts, Mildred (H)
Rapid City
*Krieger, Florence I. (H)

Tennessee
Knoxville
*Brown, Kenneth E. (C)
Nashville
*Banks, J. Houston (C)
Clement, Mary Dean (C)
*Wren, F. Lynwood (C)
Wren, Mrs. F. Lynwood (V)
Memphis
*Kaltenborn, H. S. (C)

Texas
Dallas
*Holder, Lorena (E)
San Marcos
*Bernhard, Ida M. (H)

Virginia
Charlottesville
*Lankford, Frank G. (C)
*Schuder, Gladys S. (H)
Stein, Maurice H. (St)
Farmville
*Phillips, Josephine M. (C)
Richmond
*Archer, Allene B. (H)
*Turner, Doris M. (J)

Washington
Seattle
*Roudebush, Elizabeth J.

West Virginia
Huntington
*Barron, James J. (C)
*Nozum, Lawrence (C)
Kenova
*Adkins, Julia (H)

Montgomery
*Nolan, Irene A. (C)
Morgantown
*Dorsey, Catherine (H)
*Wilt, May L. (C)
St. Albans
*Lynch, Kathryn W. (H)

Wisconsin
Appleton
*Duling, Hazel D. (H)
*Webb, Genevieve (J)
Beloit
*Fish, Kenneth R. (J)
*Miller, Hilda E. (J)
*Sward, Dorothy (J)
*Sweitzer, Matilda (J)
*Thompson, Orpha C. (H)
Columbus
*Huff, Marcia (H)
Fond du Lac
*Hansen, Estelle G. (J)
Janesville
*Bartelme, Viola B. (H)
*Davis, Marjorie H. (H)
Kaukauna
Feller, Mildred M. (H)
Kenosha
Andrews, Ona
Baar, Florence (H)
*Caswell, Florence E. (H)
*Goerz, Lydia R. (H)
*Griffin, Tella F. (H)
Novack, Catherine B. (H)
Phillips, Lucile (J)
*Schleck, Harriet H. (J)
La Crosse
*Sister M. Michtildis (C)
*Sister M. Zelma (H)
Lake Geneva
*Rassach, Cassie (H)
Madison
*Brown, John A. (H)
*Marris, Florence (H)
*Mayor, John R. (C)
Markesan
*Junker, Elmer S. (H)
Junker, Mrs. Elmer S. (V)
Menasha
*Walker, Carol (H)
Milwaukee
Bickler, Peter (E)
*Joseph, Margaret (H)
Jones, Arnold P. (C)
*Overn, Orlando E. (C)
*Sister Mary Felica (C)
Sister Mary Petronia (C)

*Weisbecker, Frances (C)
Neenah
*Anderson, Thora (J)
Racine
*Baird, H. Grace (J)
*Barry, Mary R. (J)
*Becker, Estelle R. (J)
*Colt, Ruth C. (J)
*Cragg, Maude E. (J)
Ewers, Benjamin J. (H)
*Neitael, Anne L. (H)
*Potter, Mary (Su)
Root, Dorothy A. (H)
*Shaw, M. Esther (H)
*Smith, Lucy E. (J)
Stelter, Gerhardt, (J)
*Tubbs, Charles L. (H)
*Weltman, Harriet C.
Shawano
*Mielke, Sarah J. (H)
Superior
*Person, Ruth E. (H)
*Wolfinger, Marguerite E. (H)
Waukesha
Lawler, Myrtle K. (J)
*Meadows, Paul E. (C)
Wauwatosa
Falls, Florence (E)
*Striegl, Margaret A. (H)
West Allis
*Helstern, Claire (H)
*Tarbell, Helen I. (H)
West Bend
*Neff, Lois M. (H)
*Schreiner, Caroline A. (H)

Canada
Ontario
Toronto
Gilmore, Orie A. (H)
Owen Sound
*Hinchley, John M. (C)
Kingston
Miller, Norman (C)
Port Colborne
Pollock, Carman E. (H)
New Market
*Rourke, Robert (C)
*Sharp, J. Norman (C)
Totton, H. Edwin (J)
Sault Ste Marie
Irving, Jessie C. (C)
Wright, May C. (E)

ATTENDANCE RECORD

State	Members	Guests	Total	State	Members	Guests	Total
Albama	1	0	1	Kansas	3	6	9
California	3	0	3	Kentucky	7	1	8
Colorado	11	1	12	Louisiana	4	1	5
Connecticut	1	0	1	Maryland	12	9	21
Delaware	1	0	1	Massachusetts	11	3	14
District of Columbia	3	1	4	Michigan	37	31	68
Florida	6	0	6	Minnesota	17	3	20
Georgia	4	0	4	Missouri	4	2	6
Chicago	132	226	358	Montana	1	0	1
Illinois	221	206	427	Nebraska	3	1	4
Indiana	77	30	107	New Hampshire	3	0	3
Iowa	25	11	36	New Jersey	4	0	4

State	Members	Guests	Total
New York	21	8	29
North Carolina	2	0	2
Ohio	44	5	49
Oklahoma	5	1	6
Pennsylvania	13	6	19
South Dakota	1	1	2
Tennessee	4	2	6
Texas	2	0	2
Virginia	5	1	6
Washington	1	0	1
West Virginia	7	0	7
Wisconsin	47	14	61
Canada	4	5	9
Totals	747	575	1322

Syracuse, New York Annual Mathematics Contest, 1950

By Gertrude Lohff
High School, Syracuse, New York

For the past four years, on the last Thursday in May, Syracuse student members of Pi Omicron, honorary mathematics society, have sponsored a competitive test open to all Onondaga County high school students who are taking or have completed plane geometry. Members of the society issue the invitations, explain the contest to interested high school pupils, and assume responsibility for publicity in newspapers and radio broadcasts.

The 120-minute test consists of 120 short original arithmetic, algebra, and geometry questions. The Syracuse Board of Education approves the test, prints it, and furnishes the paper for the competition. Students taking the test are given numbers so that their identity is not revealed until after the papers have been graded and the winning papers determined. Students of the sponsoring society administer, proctor, and grade the test.

The eighteen prizes range from $5 to $75 for a total of $222. Three additional prizes are awarded to the three highest students from the school that enters the greatest number of contestants. Each contestant pays an entrance fee of twenty-five cents and the sponsoring society raises the rest of the money by sponsoring a movie, a dance, and cookie and candy sales. This year a parents' organization gave $30.

Following the 1950 contest the supervisor of mathematics of the Syracuse public schools invited the winners and their parents to a party in the Hotel Syracuse. At that time prizes were awarded. The $75 first prize was awarded to a senior student from a small school in the suburbs who had won the fourth prize last year. This student had 89 correct answers out of the possible 120. Students from twelve different high schools received prizes. The fact that next year a test will also be given for junior high school students is an indication of the popularity and success of the contest plan.

The National Science Foundation

In the spring of 1950, President Truman signed the bill passed by Congress providing for the establishment of The National Science Foundation. It will have large powers "for the promotion of basic research and education in the sciences."

The Bulletin of the American Mathematical Society for July 1950 carries a report on the recommendations made by The Policy Committee for Mathematics to the newly established National Science Foundation. (The Policy Committee for Mathematics is composed of representatives of The American Mathematical Society, The Mathematical Association of America, The Institute of Mathematical Statistics, and the Association for Symbolic Logic.) Two of these recommendations should be of particular interest to members of the National Council:

"The Foundation should seek additional means of discovering and fostering promising mathematical talent among young students, particularly those who, for geographic or other reasons, may have only poor opportunities for discovering this talent for themselves. In particular, the Foundation should explore the desirability of publishing, or assisting the Mathematical Association to publish, a periodical for the benefit of secondary school students in order to stimulate and direct younger students showing definite mathematical ability and interest.

"The Mathematics Division of the National Research Council, with financial assistance where needed from the National Science Foundation, should sponsor studies of the quality of teaching and the curriculum in mathematics at the secondary school level throughout the country, and take steps to bring whatever changes are indicated as necessary by the study. Among other things, study should be made of the desirability of the development of visual and auditory aids to teaching. The writing of sound mathematics books at the secondary school level, and of good texts for undergraduates should be encouraged."

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